

TRANSITION TO WAVY VORTICES FOR INTERMEDIATE VALUES OF THE RADIUS RATIO

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Abstract

The transition from Taylor to wavy vortices is revisited for parameter values in the range of new laboratory experiments [1]. The dependence of the critical Reynolds number with the axial wavelength of the Taylor vortices is obtained for azimuthal wave numbers from 1 to 5, and for five different values of the radius ratio. We show how islands of stable Taylor vortices above the transition to wavy vortices form.

Introduction

The Taylor-Couette system can exhibit a multiplicity of stable solutions for a given value of the parameters, once the basic Couette flow becomes unstable. Jones ([2] and [3]) calculated the transition curves from steady axisymmetric Taylor vortices to wavy vortices when both have an axial wavelength of twice the gap between the cylinders, the outer cylinder is at rest and for values of the radius ratio from 0.6 to 1. His results illustrate the complex behavior of these transitions when different radius ratios and azimuthal wave numbers are taken into account.

The initial wavelength of the Taylor vortices is also an important parameter, which affects the boundaries of the secondary instabilities, as was demonstrated by Mullin and Benjamin [4] and Lorenzen et al. [5]. We examine here the onset of wavy vortices for a wide range of axial wavelengths and for systems with five different radius ratio between 0.72 and 0.8. We compare our numerical results with those of Jones [2], and with the experimental works of Burkhalter and Koschmieder [6] and Lim et al. [1]. The latter authors studied the effect of the angular acceleration on the critical wavelength of the bifurcated flow and found that stable vortices above the quasi-static transition to wavy vortices can be obtained. These vortices have axial wavelengths shorter than those obtained after a quasi-static transition from Couette flow. We

show here that these solutions are connected with the standard Taylor vortices and that they could also be obtained quasi-statically, for certain values of the radius ratio, if a mechanism of modifying the axial wavelength is available.

Methods

We have adopted a formulation based on scalar potentials in which the velocity field is divergence-free. It is fully detailed in [7] and [8]. We have assumed infinite cylinders and periodic solutions in the axial direction with axial wavelength λ . The outer cylinder is at rest. In order to solve the Navier-Stokes equations for the potentials we have used pseudo-spectral methods (Canuto et al. [9]).

As the Taylor vortex flow is stationary it can be computed using continuation methods varying different parameters (Kuznetsov [10]). The linear stability of the computed Taylor vortices has been studied considering non-axisymmetric perturbations of the same axial periodicity. The corresponding eigenvalue problem can be split into two parts by separating the potentials into its even and odd parts in the vertical coordinate z . A detailed study of the parity of their terms shows that the system can be separated into two kinds of eigenvalue problems. One of them corresponds to transitions to wavy solutions in which the boundaries oscillate. The eigenvalue problem has also been discretized by using the same pseudo-spectral method used to calculate the Taylor vortices. The code we

have developed studies the stability of each solution obtained during the continuation process and stops when the real part of the leading eigenvalue is zero. The details about the implementation of the spectral methods and the resolution employed are provided in [8].

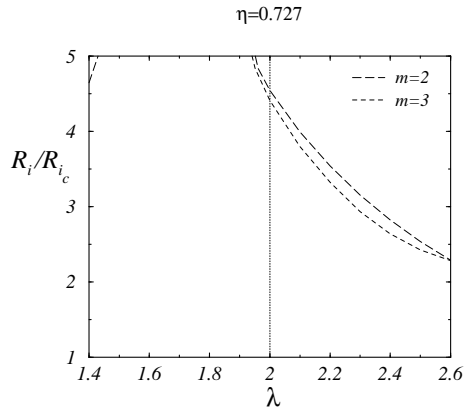


Figure 1: Neutral stability curves for the transition to wavy vortices. $\eta = 0.727$.

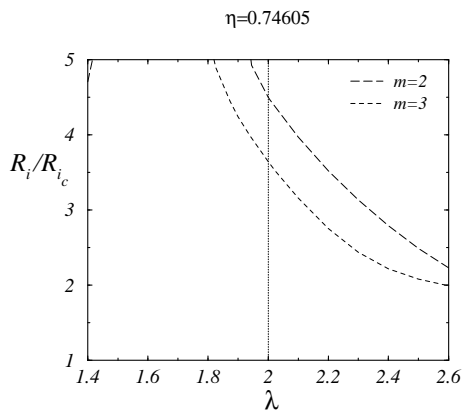


Figure 2: As for Fig. 1 with $\eta = 0.74605$

Results

The plots in Figs. 1 to 5 show the stability boundaries for the onset of wavy vortices of azimuthal wave number m from 1 to 5 in the parameter plane $(\lambda, R_i/R_{i_c})$. R_{i_c} is the critical inner Reynolds number for the transition from Couette flow to Taylor vortices. The values of radius ratio in each plot are $\eta = 0.727, 0.74605, 0.7651, 0.78415, 0.8032$, respectively. The former and the latter have been considered in order to compare them with experimental results ([6] and [1] respectively). The intersections of each vertical dotted line at $\lambda = 2$ with the neutral curves have been used to compare our

results with Jones [3] calculations. They are in very good accordance.

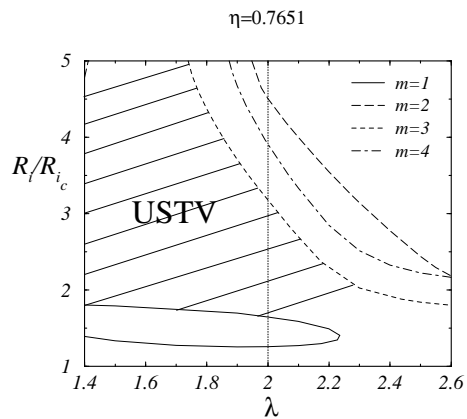


Figure 3: As for Fig. 1 with $\eta = 0.7651$

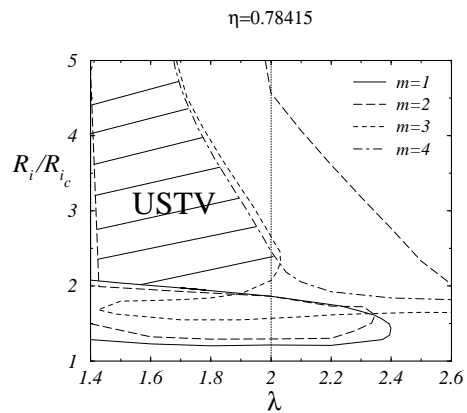


Figure 4: As for Fig. 1 with $\eta = 0.78415$

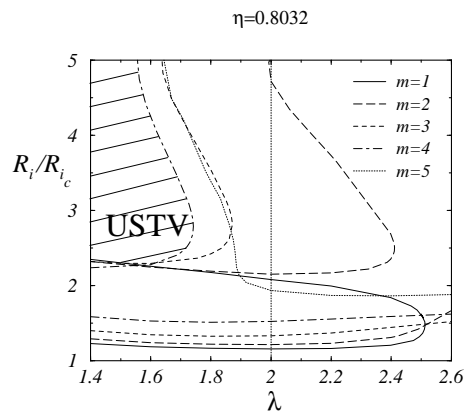


Figure 5: As for Fig. 1 with $\eta = 0.8032$

For values of $\eta = 0.727$ and $\eta = 0.74605$ the transition to wavy vortices is, for $\lambda < 2$, above the range of the Reynolds numbers of the plots leaving a wide region of stable Taylor vortices. Burkhal-

ter and Koschmieder [6] obtained these kinds of low- λ solutions in a Taylor-Couette system with $\eta = 0.727$, even for R_i/R_{ic} up to 7. By further increasing η , new transition curves appear with azimuthal wave numbers up to $m = 5$ (see Fig. 3 to 5). Some of them have folds which allows the existence of a region of stable Taylor vortices which has been dashed and labelled as USTV. In the case $\eta = 0.7651$ the vortices in the dashed region could be obtained quasi-steadily from the standard squared Taylor vortices following a suitable path, if a mechanism of varying the axial wavelength is available. Fig. 4 shows how the stability curves have separated the region of stable Taylor vortices in two disconnected components. Finally, Fig. 5 shows that the upper region of stable Taylor vortices (USTV) is now well separated from the lower region by all the transition curves, and it is confined to lower axial wavelengths.

In a recent paper, Lim et al. [1] found a regime of stable Taylor vortices with axial wavelengths shorter than those obtained quasi-steadily, in a system with radius ratio $\eta = 0.8032$. They refer to it as secondary Taylor vortex flow (STVF) and it was been found when the acceleration (dR_i/dt) was above $2.2s^{-1}$. If the acceleration is lower, wavy vortices are obtained in the same range of Reynolds numbers. The USTV region of Fig. 5 explains, at least qualitatively, the coexistence of the two regimes (Taylor and wavy vortices) observed in the experiments of Lim et al. [1]. Although the infinite cylinder approximation we use could not reproduce the conditions of the experiments, many of the experimental secondary Taylor vortex flow (STVF) obtained are inside the region USTV.

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