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Bifurcation and Application of Dean Vortex Flows

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Abstract

The stability and behavior of centrifugally-induced Dean vortex flow was analyzed and used to determine the optimum operating conditions for mass transfer in membrane filtration of suspensions. Magnetic resonance flow imaging using flow encoding with spin warp imaging was used in three dimensions to measure velocity profiles and the dynamic behavior of Dean vortices in curved tube flow. Experimental measurements were compared with numerical simulations obtained from the solution of Navier-Stokes and continuity equations using a commercial finite element package. Effects of flow rate and geometry such as the tube radius, a, and the ratio of the tube radius to the radius of curvature, r_C, on the stability of Dean vortices were studied. Twisting and bifurcation (i.e. break-up into multiple vortices) of vortices increased with increasing flow rate and increasing radius ratio, (a/r_c). A six-vortex pattern was measured experimentally and predicted numerically. Additional wall shear rates due to Dean vortices, were estimated from velocity measurements in the cross-sectional plane and used to advantage in membrane filtration of yeast and other suspensions. A phase diagram was constructed to establish conditions for the existence of two, four or six vortices as a function of flow rate and curvature. Experimental observations were compared with numerical results obtained from three types of finite element grids. The full-tube grid without symmetry planes was most predictive for vortex bifurcation, while the pseudo-cylindrical full-tube grid with a plane of symmetry gave best results for shear rates. Numerical results agreed qualitatively with the MRI measurements.

Introduction

The stability and behaviour of centrifugal vortices is of fundamental and applied interest [1,2]. In this study we endeavor to show how understanding the stability of Dean vortices during flow around a curve can be used to great advantage for membrane filtration of particulate suspensions.

With our long-term interest in fluid flow of particulate suspensions in membrane filtration modules containing opaque hollow fiber or spiral wound flat sheet membranes, magnetic resonance flow imaging (MRFI) provides a non-invasive and facile method to obtain three dimensional flow fields without the limitations of transparency. Our group has pioneered the use of MRFI for studying flows in hollow fiber membrane modules [3,4]. This technique can thus be used effectively to study the 3-D behavior of flow in curved tubes.

The work reported here follows our earlier effort and has the goal of summarizing the behavior of flow in a curved tube with centrifugal instabilities (Dean vortices) through experimental observation using MRFI and numerical simulation [5,6]. We also demonstrate how such instabilities can be used to significantly improve mass transfer and hence membrane filtration performance [7,8].

Methods

Experimental: The NMRFI system consisted of a 2 T magnet, equipped with gradient coils, in which a glass curved U-tube is placed horizontally. Water doped with 4 g/L copper sulfate ($CuSO_4$) was pumped (MDR-30T, Eastern/Iwaki Co., CT) from a 45 L reservoir through a rotameter (Model N092-04ST, Cole-Parmer, IL) to the U-tube. The data were collected on a General Electric 2T CSI imaging spectrometer using a SUN workstation. A three-dimensional pulse sequence described in detail by Chung, et al. [7] was used to obtain the longitudinal as well as the transverse velocities. The spatial and velocity resolutions of this system were approximately 60 µm and 0.1 cm/s. Five U-tubes of radii of curvature ranging from 0.084 to 0.317 were used. See Table 1.

Table 1: Dimensions of glass U-tubes [6]

Tube radius, a (mm)	Radius of curvature r_c (mm)	Radius ratio (a/r_c)
4.15	49.5	0.084
3.90	25.4	0.154
9.32	48.52	0.192
4.18	16.25	0.258
8.02	25.30	0.317

Numerical: The geometry of the U tube was described mathematically as a circle sweeping along a curved centerline with the plane of circle always perpendicular to the sweeping direction. The field was discretized into a collection of control volumes. The governing (N-S) differential equations were approximated by a set of algebraic equations on this collection, and this system of algebraic equations was then solved to produce a set of discrete values, which approximated the solution of the partial differential system over the field. Three types of computational grids have been used in this study: (a) a pseudo-cylindrical half-tube grid with an artificial radius of 1/1000 of the actual tube radius (to circumvent the singularity at the tube center), (b) a pseudo-cylindrical full-tube grid with an artificial radius of 1/1000 of the actual tube radius and (c) a rectangular grid for the full tube without symmetry planes. Simulations were carried out for Reynolds numbers ranging from laminar flow (20) up to turbulent flow (~ 5000).

Results

Vortex behavior: A detailed analysis of the behaviour of the axial velocity contours (Fig. 1) and the transverse velocities (not shown) are analysed as a function of radius ratio, tube radius and flow rate.

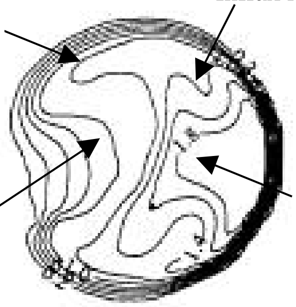

Figure 1: Typical MRFI results for contours of axial velocities as a fraction of the average axial velocity vectors [6].

First, we will demonstrate that there is correspondence in behavior between axial and transverse velocities, in terms of twisting and bifurcation. Then, through measuring the morphogenesis of the twisting and bifurcation of Dean vortices with increasing flow rate using MRFI, we found that the primary vortices twist *against* the centrifugal force. At similar Dean numbers, twisting was found to be more prevalent in tubes of smaller diameter, while bifurcation of the vortices was more extensive in tubes of larger diameter. On the other hand, twisting and/or bifurcation occurred at lower flow rates as the radius ratio increased. The twisting/bifurcation increased with increasing flow rate. At least for tubes of smaller diameter, the twisting/bifurcation decreased in the transition regime and vortex patterns similar to those at low flow rates were re-established at high flow rates. Next, bifurcation of the standard vortex doublet, to four and then to six vortices, was obtained for the tube of the highest radius ratio (0.317). Axial velocity profiles at these flow rates showed influx regions, that is, velocity contours moved in opposite directions. The parameter Re (a/r_c) for the flow rate at which the four-vortex pattern was fully established was found to be nearly constant for all the radius ratios. The same parameter at the onset of the four-vortex instability showed greater variation. Both these values were much higher than those reported in literature.

For mass transfer applications such as membrane filtration, the mean shear rate at the solid wall of the flow channel is critical and should be maintained as high as possible. Wall shear rates were estimated from transverse MRFI velocity measurements. These shear rates increased with increasing flow rate, passed through a maximum, decreased to a minimum and then increased again (Fig. 2). The maxima corresponded to the onset of twisting/bifurcation to the four-vortex pattern, and the minima corresponded to the fully established four-vortex pattern. Similar “zigzag” behavior (with maxima and minima at about the same Dean number) was observed for filtration flux with axial flow rate suggesting that vortex stability strongly influenced filtration rates and emphasizing the need to understand vortex behavior. Thus, primary vortices were shown to move against the centrifugal force, similar to the experimentally observed results.

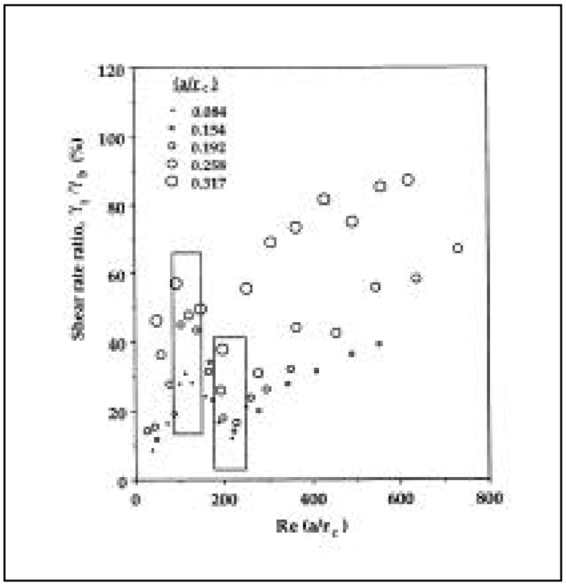

Figure 2: Effect of Re(a/rc) on normalized transverse shear rates from MRFI measurements [6].

With respect to the numerical results, twisting of the vortices was not observed for any of the grids. Simulations were performed with increasing as well as decreasing Reynolds numbers. No hysteresis was found between simulations with increasing and decreasing Re. This contradicted the results reported in literature. Friction factors calculated from numerical pressure drop data supported the hypothesis that the wide-gap theory developed for flow in curved slits can be used for flow in curved tubes to calculate the critical Reynolds number. The best agreement was found for the full-tube grid without symmetry planes. Numerically obtained wall shear rates increased monotonically with increasing Re, as opposed to the wavy curve found experimentally. The best agreement was found for the pseudo-cylindrical grid for the full tube. Numerically obtained axial velocity contour profiles and transverse velocities were compared with those obtained experimentally by NMRFI. The best agreement was found for the full-tube grid without symmetry planes. Influx regions for the axial velocity contours and the six-vortex pattern were obtained.

Mass Transfer Improvement: Pressure-driven membrane processes were first developed in Germany to filter surface waters contaminated with bacteria during the First World War. Later, they were used to separate uranium isotopes in the 1940s for the Manhattan Project and in the 1960-70s for desalting of brackish and seawaters. Early module designs such as flat sheet, tubular, hollow fiber and spiral wrap were used in conjunction with extensive pretreatment to remove most foulants. Only when these processes began to see application in the biotechnology and other industries in the 1980-90s, where macromolecular solutions and particulate suspensions needed to be filtered, did the problem of fouling (i.e. pore plugging and cake build-up) become a major problem. Various approaches have been used to address the limitations posed by fouling. These include modifying the chemical properties of the membrane surface so as to minimize solute-membrane interactions and improved fluid mechanics and module design for reducing solute concentration and deposition on the membrane. Clearly, an attractive option is to use centrifugal vortices such as Taylor [9] and Dean vortices to increase wall shear rate and hence reduce fouling through increased back-mass transport away from the membrane.

Our group has patented the use of Dean vortices in curved slit and tubular flow to reduce membrane fouling during microfiltration and ultrafiltration of several typical biotechnology feed suspensions (*E. coli*, yeast broth and mammalian cell culture fluids), model solutions (of bovine serum albumin, dextran and polyethylene glycol) and model suspensions (colloidal polystyrene and silica spheres) [10]. The approach has also been applied to reducing salt build-up during nanofiltration of aqueous solutions of salts. The vortices depolarize the build-up of suspended particles such as yeast cells or dissolved ions at the membrane-solution interface and allow for increased membrane permeation rates and increased solute permeation. Encouraging results for microfiltration, ultrafiltration and nanofiltration suggest that such vortices should be used in

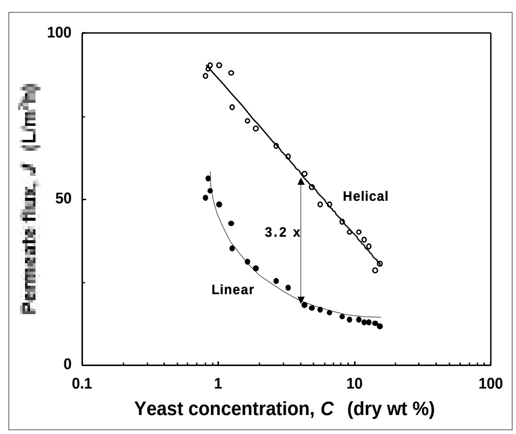

Figure 3: Effect of feed concentration of yeast on permeate flux during filtration of a yeast suspension with linear.and helical membrane modules. Transmembrane pressure was 14 kPa and Reynolds numbers were 848 (initial) and 382 (final) [7].

commercial membrane modules. At relatively low azimuthal flow rates, flux improvements from 20 to 400 % for flow with over that without Dean vortices were observed. One example in which suspended yeast particles are used as feed to both a helical filter (with Dean vortices) and a linear control filter (without vortices) is shown in Fig. 3. Negotiations to commercialize this technology are underway.

Conclusions

Velocity profiles for flow in curved tubes were measured using nuclear magnetic resonance flow imaging (NMRFI). Numerical solutions of the Navier-Stokes equations were obtained with a finite volume method to approximate the experimental results. Measured results showed that twisting/bifurcation increased with increasing flow rate and increasing radius ratio, (a/r_c). A six vortex pattern was obtained both experimentally and numerically. Additional wall shear rates due to Dean vortices, were estimated from velocity measurements in the cross-sectional plane. Experimental observations were compared to numerical results obtained from three types of finite element grids.

Successful application of Dean vortices to membrane filtration have exhibited excellent improvement in performance sometimes as high as 400 % better than current linear commercial filters.

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