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EFFECT OF AXIALLY-PERIODIC INNER-CYLINDER RADIUS ON STEADY AXISYMMETRIC COUETTE AND TAYLOR-COUETTE FLOW

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Abstract

We report computations of axisymmetric steady solutions of the Navier-Stokes equations for flow in an annulus, driven by rotation of an inner cylinder with axially-periodic radius. The results are compared to those for the constant-radius (standard Taylor-Couette) case, and it is seen that the bifurcation structure, while similar, has important differences. We also discuss the likely stabilization of the steady axisymmetric flow by axially-periodic radius variation, as well as the potential usefulness of that technique in stabilizing steady axisymmetric Taylor vortex flow at Taylor numbers beyond the values at which wavy Taylor vortex flow would set in for the constant-radius case.

Introduction

One of the simplest geometric perturbations of Taylor-Couette flow is when one of the axisymmetric boundaries is allowed to have an axially-periodic radius. This problem is of interest in several contexts.

Besides work on the effect of localized radius variation [1-2], the only work on extended (i.e., periodic or random) radius variation known to us is experimental work by Ikeda and Maxworthy [4], Painter and co-workers [5-10], and Smits, Auvity & Sinha [11].

Here, we present a computational investigation of the flow between a stationary outer circular cylinder of constant radius r_o and a rotating inner surface of revolution whose radius varies sinusoidally in the axial direction about a mean value r_i, focusing on the multiplicity of the solutions as a function of Taylor number for several different amplitudes. The ratio of the mean inner-cylinder radius to the outer-cylinder radius is held constant at 0.5, corresponding to the "wide-gap" case, in which steady axisymmetric Taylor vortices persist to about ten times the first critical Taylor number for a constant-radius inner cylinder.

Numerical Methods

The full Navier-Stokes equations were solved numerically using finite-element methods. A consistent penalty method was used to satisfy incompressibility. Quadrilateral elements with quadratic velocity and linear pressure interpolation were used.

Discretization of the equations leads to a quadratically nonlinear algebraic equation system which is solved by Newton-Raphson iteration. The computations were performed at a series of increasing spatial resolutions, with the analytical approximation of Taylor-Couette flow for the small gap in a circular annulus being used as the initial iterate for the lowest resolution. The initial iterate at each resolution was obtained by interpolation from the converged solution at the previous resolution.

Results

For $\eta = r_i/r_o = 0.5$, we begin by considering the standard (constant-radius) Taylor-Couette case, and compare our computational results to work of previous investigators. From the Couette branch, on which the dimensionless torque is a linear function of Taylor number (Ta), other branches on which the torque increases, generally, superlinearly with Ta. These branches bifurcate from the Couette branch at Taylor numbers greater than the first critical value. The branch which bifurcates from the Couette branch at the smallest Ta has the highest torque up to $Ta \sim 210$, beyond which the highest-torque branch is the second one to have bifurcated from the Couette branch.

As the amplitude of the sinusoidal radius variation increases from zero, the primary Couette-like branch corresponds to flows which are progressively less purely azimuthal. Concomitantly, the nature of the bifurcation from the primary branch changes. For sufficiently small amplitude, the torque on the primary branch is a nearly linear function of Ta up to and slightly beyond the Ta at which a branch of solutions bifurcates from the primary branch at a Ta similar to that for the standard case. Beyond that Ta, however, the torque on the primary branch increases superlinearly with Ta. At still higher Ta, another solution branch bifurcates from the primary branch. On this branch, the torque-*Ta* relation follows a linear extrapolation of the low-*Ta* primary branch.

At higher amplitudes of the radius variation, the bifurcation structure becomes more complicated. At still higher amplitude, only a single branch is found.

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