SPIRAL AND WAVY VORTEX FLOWS IN SHORT COUNTER-ROTATING COUETTE-TAYLOR CELLS

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Abstract

Vortical flow in a confined rotating system is a problem with many industrial and fundamental applications. In this study we investigate the case of cylindrical Couette flow in a finite-length cavity with counter-rotating walls via direct numerical simulation using a three-dimensional spectral method. We consider aspect ratios ranging from L = 5 to L = 6. Two complex flow regimes, wavy vortices and spirals, occur with similar appearance to those found experimentally for much larger aspect ratios. The endwalls appear to damp and stabilize the flow as the aspect ratio is reduced.

Introduction

The Taylor–Couette system of shear flow between concentric cylinders provides valuable insight into the stability of flows in rotating systems and the interaction of various vortical structures. The Taylor-Couette configuration with a fixed outer cylinder has been widely investigated experimentally and numerically since Taylor's work in 1923. Experiments and computations have established that the flow changes according to the following scenario: stable Couette flow, axisymmetric Taylor vortices, wavy vortices, modulated wavy vortex flow, and turbulent vortices.

The rotation of the outer cylinder in addition to the inner cylinder results in a variety of other flow regimes : wavy inflow and outflow, wavelets, twisted vortices for co-rotating cylinders; interpenetrating spirals, wavy interpenetrating spirals, spiral turbulence for counter-rotating cylinders [1]. At the present time, most numerical studies have been undertaken for a fixed outer cylinder with only a few for the co- or counter-rotating cases.

For short cylinders with the outer cylinder fixed, the endwalls play a significant role [2]: the transition to non-wavy vortical flow occurs at a lower rotational speed and the subsequent transition to wavy vortical occurs at a higher rotational speed than in the case of infinitely long cylinders. The pattern of the flow can be strongly influenced by the confinement between the two endwalls. The key parameter is the aspect ratio, $L = 2h/d$, where $2h$ is the distance between the endwalls, and d the width of the annular gap.

To our knowledge, no experimental or numerical results are available for short differentially rotating cylindrical Couette systems where endwall effects are likely to be important. This lack of data has motivated our study of finite-length counter-rotating flows by means of a spectral Chebyshev-Fourier method. This method has been shown to be effective for studying complex phenomena in rotating flows [3].

Geometry and numerical method

The system is an annular cavity bounded by two concentric cylinders of inner and outer radii r_i and r_o, respectively, that rotate independently at W_i and W_o. The endwalls at $z^* = \pm h$ rotate with the outer cylinder at W_o.

The flow is described by the incompressible 3D Navier-Stokes equations written in cylindrical coordinates (r^*,z^*,q) in an absolute frame of reference according to velocity-pressure formulation. Characteristic parameters are the Reynolds numbers $Re_i = W_i r_i^* d /n$ and $Re_o = W_o r_o^* d /n$, the radius ratio $h = r_i^*/r_o^*$, and the aspect ratio $L= 2h / d$, where $d = r_o^* - r_i^*$. The scales for the dimensionless variables of space, time, and velocity are h, W_o^{-1}, $W_o r_o^*$, respectively. The dimensionless radial and axial coordinates are $r' = r^*/h$, $r' \in [r_i^*/h; r_o^*/h]$, and $z = z^*/h$, $z \in [-1; 1]$. The use of Chebyshev polynomials requires r' to be normalized on [-1; 1], so that the normalized radius is $r = r'L - (r_o^* + r_i^*) / d$.

On the boundaries u and w obey the no-slip condition. The singularity at r = -1 between the endwall rotating at W_o and the inner cylinder rotating at W_i is regularized by means of an exponential function.

The solutions are computed with a pseudo-spectral Fourier-Chebyshev collocation method. The time scheme is semi-implicit and second order. It consists in the combination of the second-order backward implicit Euler scheme for the time term, an explicit Adams-Bashforth scheme for the non linear terms, and an implicit formula for the viscous diffusion term :

$$\frac{3V^{n+1}-4V^{n}+V^{n-1}}{2dt}+2(V^{n}\cdot\nabla)V^{n}-(V^{n-1}\cdot\nabla)V^{n-1}$$
$$=-\nabla p^{n+1}+\frac{1}{Re}\Delta V^{n+1}+F^{n+1}$$

The process of the velocity-pressure coupling is performed using an improved projection algorithm [4]. The approximation of the exact solution $\Psi=(u,v,w,p)$ is given by:

$$\Psi_{NMK}(r,z,q,t)=\sum_{n=0}^{N}\sum_{m=0}^{M}\sum_{p=-K/2}^{K/2-1}\widehat{\Psi}_{nmp}(t)T_n(r)T_m(z)e^{ipq}$$

where $(r,z,q)\in[-1;\ 1]^2\times[0;\ 2\pi]$, T_n, T_m are Chebyshev polynomials, and $\widehat{\Psi}_{nmp}$ are the Fourier coefficients.

Wavy vortex flow

Setting the Reynolds numbers to Re_o = -250, Re_i = 750, results in the wavy vortex flow (WVF) for *L*=6 shown in Figure 1, which is consistent with what is observed experimentally for *L*=30 [1]. At these values, the flow exhibits 8 rolls perturbed azimuthally by travelling waves of wavenumber *m* = 5, which is reasonably close to the 6 or 7 waves found in [1]. The dominant frequency, probably related to the travelling waves, is f / f_i = 0.4, where f_i is the rotation frequency of the inner cylinder. This is similar to the value of 0.38 found experimentally for short aspect ratios with the outer cylinder fixed, but greater than that of 0.14 found for long counter-rotating cylinders.

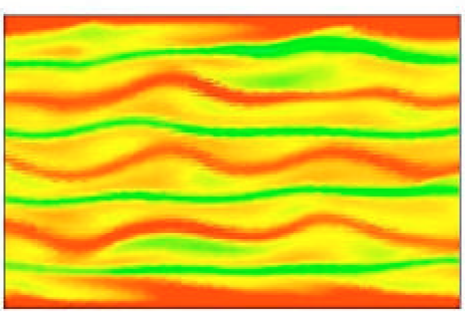

Figure 1: Three-dimensional time-dependent Wavy Vortex Flow, Re_o = -250, Re_i = 750, *L* = 6, *h* = 0.75. Instantaneous iso-values of azimuthal component of velocity in a circumferential surface (*r = 0*, θ, z), 0 £ *q* £ *p*..

The axial wavelength l/d = 1.5 is not very different from l/d =1.67 to 1.76 for WVF between long-counter-rotating cylinders at somewhat lower Reynolds numbers [1]. For *L* = 5, we found 6 vortices, giving l/d *=1.67*. Consistent with previous studies of WVF for long cylinders with the outer cylinder fixed [5], there is significant fluid transport between vortices as shown in Figure 2. The flow for *L* = 5 is very similar to that at *L* = 6, except for fewer rolls.

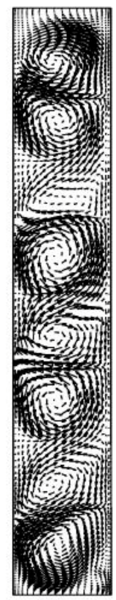

Figure 2: Three-dimensional time-dependent Wavy Vortex Flow, Re_o = -250, Re_i = 750, *L* = 6, *h* = 0.75. Instantaneous vector plot of velocity projected on an azimuthal plane (*r*, z).

Spiral flows

For Re_o = -500, Re_i = 330, and *L* = 6, an interpenetrating spiral flow results (see Figure 3). The general structure is that of two spiral vortices with opposite helicity interpenetrating each other in the entire cavity, except near the upper and lower endwalls, where only one type of spiral occurs. The upper spiral forms an angle of 9° with the horizontal plane, whereas the lower is sloped at 5°. Projection of velocity vectors on azimuthal planes shows that the spiral vortex structures are not separated radially but are confined to the unstable layer which is about 0.36*d* from the inner cylinder.

Decreasing the length of the cylinders to *L* = 5.2 while keeping the same Reynolds numbers leads to a flow that mixes both Taylor vortices and spiral structures. Taylor vortices are split due to the interaction of the spiral. For $L \leq 5$ the spiral structures vanish leaving only a Couette flow with Ekman vortical cells driven by the endwalls.

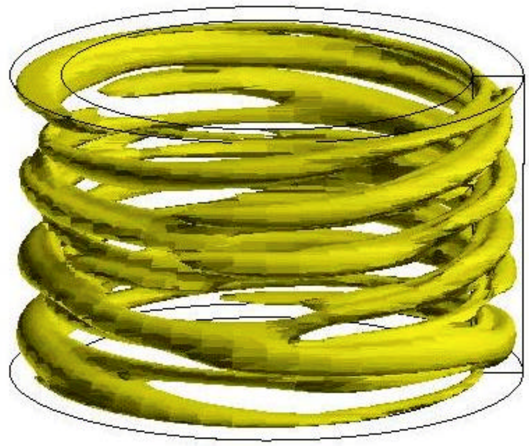

Figure 3: Three-dimensional time-dependent spiral regime, Re_o = -500, Re_i = 330, h = 0.75, L = 6. Three-dimensional displays of instantaneous isosurfaces of the radial component of velocity, $u = -1.26.10^{-3}$.

Returning to the longer annulus of L = 6, and increasing the inner cylinder speed from Re_i = 330 to Re_i = 370 results in wavy interpenetrating spirals (WIS) shown in Figure 4. The structure appears similar to that observed experimentally at slightly different Reynolds numbers [1].

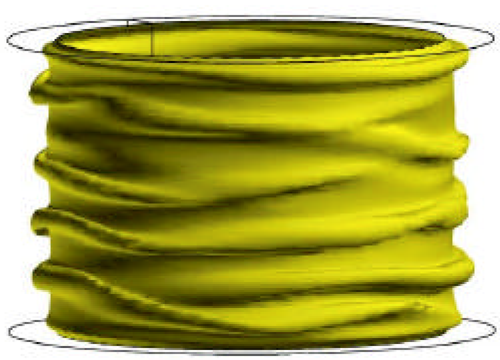

Figure 3: Three-dimensional time-dependant wavy spiral flow, Re_o = -500, Re_i = 370, h = 0.75, L = 6. Three-dimensional display of instantaneous isosurface of the azimuthal component of velocity, $v = -1.6.10^{-1}$.

Conclusion

The appearance of the flows is consistent with what found for a larger aspect ratio [1]. Moreover, the interaction between Ekman flow and Taylor flow drives the vortex structure. The effect of the endwalls is to damp and stabilize the vortical structure as L decreases. Further work is ongoing in order to determine how the flow depends on the endwall boundary conditions and Ekman vortex structure.

References

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