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MAGNETIC RESONANCE IMAGING STUDY OF TAYLOR-COUETTE-POISEUILLE FLOW

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Abstract

The Taylor-Couette problem with imposed axial through flow for a radii ratio $\eta = 0.5$ and aspect ratio $\Gamma = 16$ is investigated using Magnetic Resonance Imaging (MRI). Two MRI spin-tagging techniques were used to characterize several flow regimes in the strongly nonlinear domain in the parameter space defined by the axial Reynolds number and the Taylor number corresponding to the rotation of the inner cylinder. A fast acquisition technique (FLASH sequence) allowed the observation of unsteady regimes, while a technique with longer acquisition time but better resolution (spin-echo sequence) allowed measurements in steady flows. For the first time to the authors' knowledge, the 3-D reconstruction of the velocity field for the Stationary Helical Vortices (SHV) regime was obtained, and a spatially mixed state of SHV and Propagating Toroidal Vortices (PTV) was visualized.

Introduction

As the present workshop demonstrates, the study of Taylor-Couette flow has continued with an investigation of its variants. In this report, we face the challenge to study such flows in the strongly non-linear regime and in realistic geometrical configurations. A non-invasive cinematographic Magnetic Resonance Imaging (MRI) technique has been developed [1] for the Taylor-Couette problem with an imposed axial Poiseuille flow, referred to as the Taylor-Couette-Poiseuille (TCP) problem. The present study focuses on steady and unsteady hydrodynamic modes that emerge as the rotational speed of the inner cylinder and pressure-driven axial flow rate are varied while the outer cylinder is kept fixed. These modes constitute primary, secondary, and higher order bifurcations, which break the symmetry of the base helical Couette-Poiseuille (CP) flow and represent drastic changes in flow structure.

The study of laminar vortical structures has been well documented in the Taylor-Couette (TC) flow for an extensive range of gap sizes [2]. Of particular interest are the experimentally determined boundaries in the map of flow regimes [3] where the propagating helical vortex and stationary toroidal vortex modes become stable simultaneously and interact in the presence of the $\mathbf{O}(2) \times \mathbf{SO}(2)$ symmetry of the TC apparatus. The bifurcation diagrams in the vicinity of such a codimension-2 point indicate the possibility for coexistence (bicriticality) of these two modes for wide-gap annuli [4,5]. This presents new opportunities to study complex nonlinear interactions at relatively low rotation rates (i.e. in the neighborhood of the primary bifurcation) in variants of the TC problem, such as TCP.

The flow domain of TCP is characterized by two geometrical parameters: the ratio of the inner cylinder radius to the outer one, $\eta = R_1/R_2$, and the ratio of the length L of the cylinders to the annulus gap width ($d = R_2 - R_1$), termed the aspect ratio $\Gamma = L/d$. TCP is also characterized by two dimensionless dynamical parameters corresponding to the angular velocity of the inner cylinder (Ω) and the imposed mean axial velocity (U_z): the Taylor number, $Ta = 4\, d^4\, (\Omega/\nu)^2 \eta^2/(1 - \eta^2)$, and the axial Reynolds number, $Re = U_z d/\nu$. The majority of TCP studies concern primary bifurcations [6,7] or relatively narrow gaps [8]. However, optical [9,10] experimental studies have established the existence of various non-axisymmetric regimes with wider gaps, as seen in Fig. 1 (the nomenclature used to characterized the various flow regimes is detailed in the Results section). Motivated by the bicritical states observed in TC systems, similar behavior is also anticipated in wide-gap TCP systems. The experimental technique employed here obviates the limitations of optical diagnostics or ultrasound techniques, which extract velocities in only a portion of a meridional plane of the annular flow.

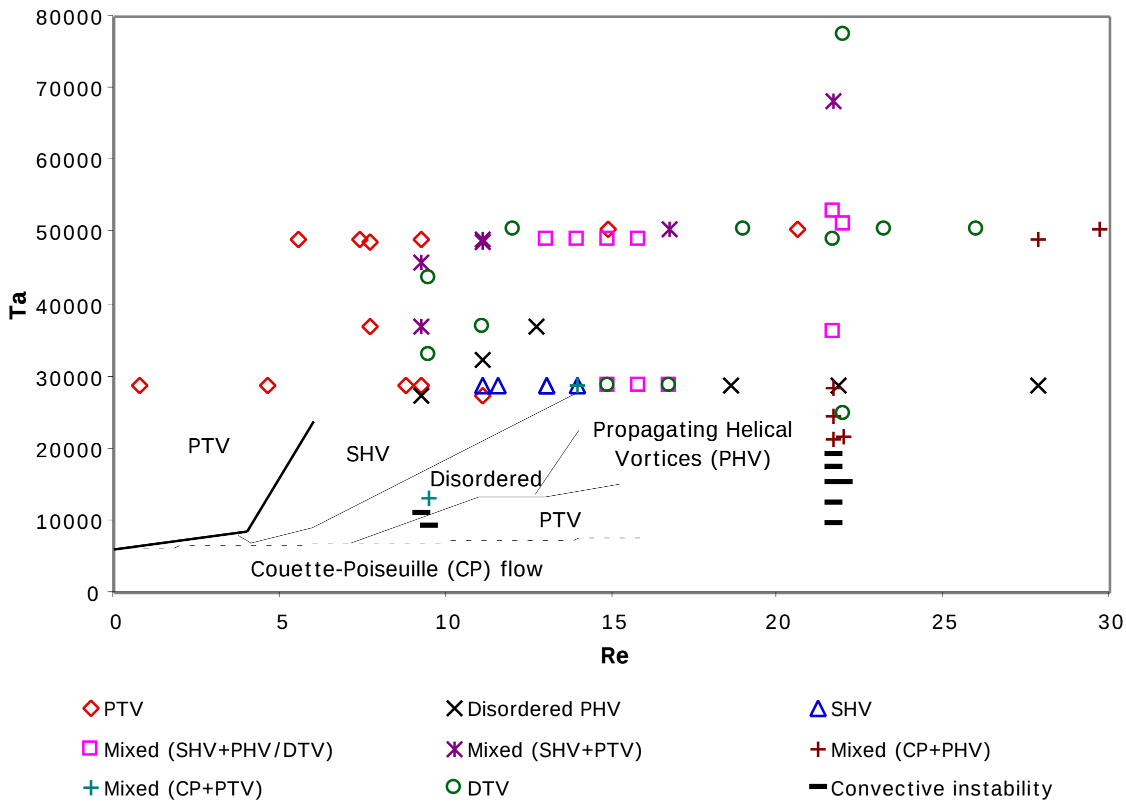


Figure 1: Flow regimes observed in TCP flow for η = 0.5 (discrete data points) and η = 0.77 (curved boundaries and flow regimes adapted from [10]).

Experimental method

The test section used in the MRI experiments was fabricated from non MRI-active materials (such as acrylics and elastomers) with η = 0.5 (inner radius R_I = 0.9525 cm) and aspect ratio Γ = 16 (length L = 15.24 cm). Two 2.54-cm-thick disk with 1-mm-perforated holes were fixed at both ends of the annulus to ensure that the flow entering and leaving the annulus was mainly axial. The axial flow was maintained by pumping water in a closed loop, while the inner cylinder was rotated via a stepper motor placed at a sufficient distance from the MRI magnet. Further details of the experimental set-up are available elsewhere [1].

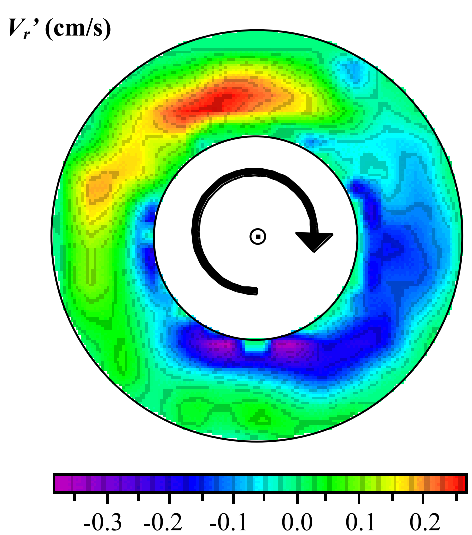
A spin-tagging technique was used to visualize the different flow regimes encountered as *Ta* and *Re* were varied (Fig. 1). Spins of the hydrogen nuclei from water molecules were tagged forming a Cartesian grid on longitudinal and transverse slices. They were then allowed to follow the flow for a fixed amount of time before they were imaged via one of two imaging techniques. A classic spin-echo imaging sequence was used mainly for steady flows since it requires an acquisition time of the order of 6 minutes for a slice with a 256 × 256 pixel resolution. A Fast Low Angle Shot (FLASH) imaging technique was used for the unsteady flows, with an acquisition time of 0.25 s for a 128 × 96 resolution [1]. Both techniques produce high contrast snapshots of streaklines evolving from the originally straight Cartesian gridlines. Such sequences allow the qualitative and quantitative study of the flow.

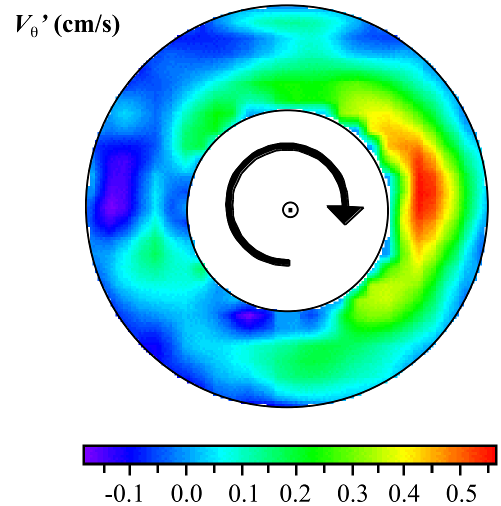
Results

The investigation of the strongly nonlinear domain in the (*Re*,*Ta*) parameter space of TCP flow allows the following two general observations. (i) A number of flow regimes could only be characterized as mixed states (cf. Fig. 1) or disordered states, such as Disordered Toroidal Vortices (DTV), Disordered Moving Helical Vortices (DMHV). These correspond to strong nonlinear competitions between coexisting stable states, where in mixed states flow regimes are stable in different parts of the annulus while in disordered states flow regimes interact in time. (ii) The boundaries between the various flow regimes were obscured by strong hysteresis phenomena encountered while spanning the parameter space.

The scope of this manuscript is limited to the flow regimes observed for two particular Taylor numbers ($Ta^{1/2}$ = 170 and 220) as *Re* is increased. The flow evolves from PTV to a spatially mixed state of SHV and PTV, via a pure SHV regime for the lower value of *Ta*. The spatially mixed state of SHV and PTV corresponds to SHV filling the annular cavity starting from the inlet, converting PTV downstream. Using the FLASH imaging sequence, a growing oscillating instability was observed in the out-flowing boundary jets between the counter-rotating of the SHV. Thus the limiting axial position between the two regimes can be interpreted as the point for which the instability has filled the whole gap between the cylinders, giving birth to a pair of toroidal vortices. This pair later grows and propagates at approximately 1.2 times the imposed axial mean flow.

An analogy between this mixed flow regime found in TCP flow and the codimension-2 state where both toroidal vortices (stationary) and porpagating helical vortices are stable in Taylor-Couette flow [4,5] was proposed, using a Galilean transformation [11]. This analogy supported a time scale analysis that showed that the average time a fluid particle takes to make a revolution around the inner cylinder equals the time it takes to translate axially by a distance of one wavelength [11].





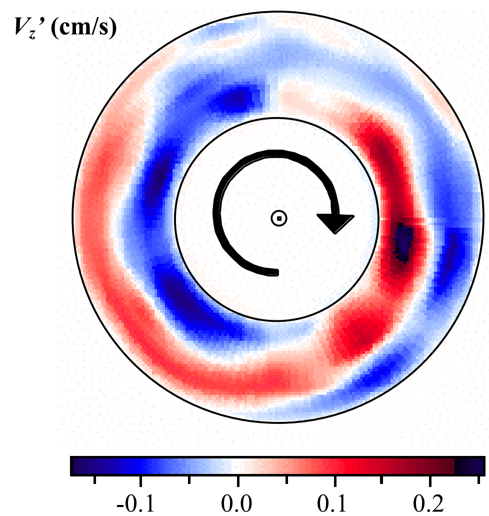


Figure 2: Reconstructed 3-D velocity field fluctuations V_r', V_θ', and V_z' in (x,y)-plane for SHV flow with Couette-Poiseuille flow as a reference flow.

The SHV flow regime was studied using the spin-echo imaging sequence. Longitudinal images revealed the helical structure of the flow as outflow boundaries on one side matched the inflow boundaries on the other, and were used to measure the axial wavelength λ (defined as the axial length of a pair of counter-rotating vortices). Transverse images taken at 5 different axial positions were nearly identical within the proper rotation. An additional slice parallel to but not containing the cylinders axis was used to determine the helicity of the SHV. MRI images for both SHV and are available elsewhere [11]. The 3D-velocity field for SHV flow was then reconstructed from the grid deformation in transverse and longitudinal sections. By convention, the z-axis corresponds to the axis of the cylinders. The x- and y-components (or r- and θ-components in cylindrical coordinates) were estimated using the displacement of the nodes of the initial Cartesian grid and the known delay before the acquisition of the deformed one. The z-component of the velocity field was determined using the displaced lines initially perpendicular to the z-axis. The helical symmetry $z = z_0 - \lambda\theta/2\pi$ was used to reconstruct the 3D-velocity field, where z_0 is the reference axial location and λ is the axial wavelength. Fig. 2 shows the three components of the velocity fluctuations, V_r', V_θ', and V_z', relative to the base CP flow field. The 3-D velocity field reconstruction allows the characterization of the transport of a passive scalar in the SHV flow, which can be validated by a direct MRI measurement.

Conclusions

MRI was used as a non-invasive visualization technique to investigate the TCP problem for the first time. It allowed the 3-D reconstruction of the velocity field for the SHV regime and the observation of two modes coexisting in different spatial location, giving birth to a spatially mixed state of SHV and PTV.

Acknowledgements

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