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Numerical investigation of the flow in wide spherical gaps

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Abstract

The basic flow of a viscous incompressible fluid in the spherical Couette system becomes unstable when the rotation of the inner sphere exceeds a critical value. The supercritical flow structures depend on the gap width. In wide gaps, spiral waves are formed. In this study, we present the results of a numerical stability analysis for a wide range of aspect ratios, and of numerical simulations of the supercritical flow state.

Introduction

We investigate the flow of a viscous incompressible fluid in the gap between two concentric spheres which is driven by the rotation of the inner sphere. The control parameters are the Reynolds number $\mathrm{Re} = R_1^2\Omega_1/\nu$ and the relative gap width $\beta = (R_2 - R_1)/R_1$, with ν the kinematic viscosity, Ω_1 the rotation frequency of the inner sphere and R_1 and R_2 the radii of the inner and the outer sphere (see fig. 1 for a sketch of the system).

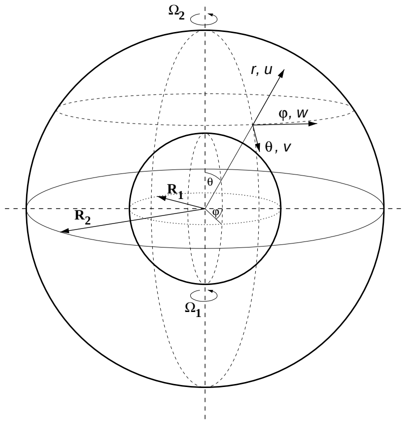

Figure 1: Principle sketch of the geometry of the spherical Couette system.

Most of the investigations, numerical and experimental as well, concentrate on the case of narrow gap widths. Recent work on the wide gap case comprises experimental stability investigations [4, 6], experimental study of the dynamics using chaos-analysing techniques [7], linear stability analysis [1] and numerical simulation [3].
The axisymmetric basic state consists of the azimuthal primary flow, which is superposed by a secondary flow in meridional direction. With increasing Re, the basic flow becomes unstable and different forms of instabilities are observed, depending on the gap size. In narrow gaps, a centrifugal instability to axisymmetric perturbations leads to Taylor vortices. In wide gaps ($\beta \gtrsim 0.25$), rotating spiral waves are observed as the first supercritical pattern; they result from an oscillatory linear three-dimensional instability of the basic state. This flow pattern and the critical values were described for the first time by Yavorskaya [8] ($\beta = 0.5$) and Belyaev [2] ($\beta = 1.0$).

Numerical Methods

For the numerical treatment of the incompressible Navier-Stokes equations, a spectral method is applied [5]. The poloidal-toroidal representation $\mathbf{u} = \nabla \times (T(r,\theta,\varphi,t)\mathbf{e}_r) + \nabla \times \nabla \times (S(r,\theta,\varphi,t)\mathbf{e}_r)$ for the velocity field is used, and the scalar fields T and S are expanded in a series of Legendre polynomials for the angular directions and Chebyshev polynomials for the radial direction. A Runge-Kutta method is implemented for the time-stepping of the coefficients, and the nonlinear terms are treated with a standard pseudo-spectral method. A modified code is used for the calculation of the stability boundaries: The linearised equations, describing the temporal development of a perturbation to a precalculated basic state, are solved with this time-dependent code.

Results

From the growth rate of the perturbations for different Re and for different given azimuthal wave numbers m of the perturbation, the critical Reynolds number Re_c and critical wave number m_c could be determined numerically for a wide range of aspect ratios ($0.28 \leq \beta \leq 3.0$), in very good agreement with and considerably extending former numerical studies ([3, 1]). Furthermore, the results are in good agreement with nearly all available experimental investigations. Figure 2 shows the marginal curves for different azimuthal modes m as function of the aspect ratio β. In the investigated regime, m_c decreases with β.

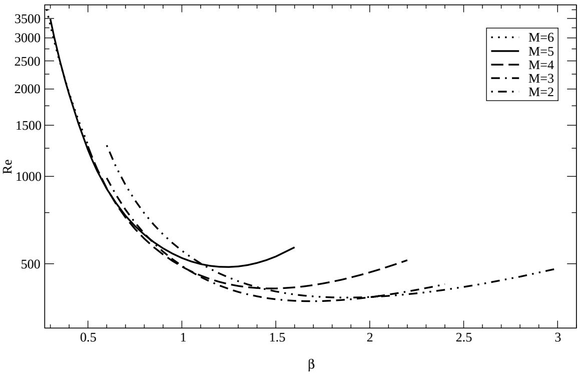

Figure 2: Neutral curves of Re for different azimuthal wave numbers m as functions of the relative gap width β.

The structure of the critical mode is shown in fig. 3 for the case $\beta = 0.498$ ($m_c = 5$); the spiral pattern in higher latitudes and the strong disturbance in the equatorial region can be seen clearly. The perturbation is equatorially antisymmetric.

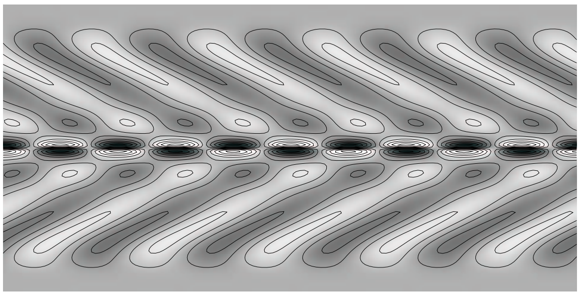

Figure 3: Contour plot of the radial velocity of the disturbance field for $\beta = 0.498$ and $\mathrm{Re} = 1260$ ($\mathrm{Re}_c = 1245$) at a radial position in the outer layer ($r = R_1 + 0.7 \cdot (R_2 - R_1)$).

For higher Re, in all experiments a mode change occurs, and the number of spirals is reduced by 1. The characteristic features of this transition can also be found in the simulations.

Conclusion

The spherical Couette flow in wide gaps was investigated numerically, and the linear stability values could be calculated in good agreement with other observations for a wide range of aspect ratios.

Acknowledgement

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