On The Nonlinear Stability Of Anisotropic MHD Planar Couette Flows

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Abstract

Nonlinear stability of the hydromagnetic Couette-Poiseuille flows in L^2-norm, to 3-D perturbations, in presence of MHD anisotropic Hall and Ion-slip currents with transport parameters β_H and β_I is investigated by the direct Liapunov functional method. It is proved that high levels of the anisotropic currents can assure asymptotic stability, according with the conditions

$$\begin{cases} \dfrac{\beta_I}{\beta_H} > 1 \\ \beta_I > 1 \end{cases}$$

Introduction

The dynamics and the stability of electrically conducting fluids in Hydromagnetics, on taking into account of the so-called anisotropic ion-slip currents [1] [2], have been subject of some investigations [3]...[9], owing the interest of these problems from the mathematical point of view, as well as for several industrial applications; for instance in MHD power generators, in technological MHD devices and processes, in Diagnostics and communications, in Flight applications, in liquid-metal MHD, in Astrophysics and Geophysics, widely studied in the isotropic case [10][11].

Because in some of these processes the imposed magnetic field H_0 is often very strong (magnetic flux density $B_0 \simeq 10T$ and magnetic permeability $\mu_e \equiv 10^{-6}H/m$, so that H_0 is about of the order 10^7) [1], the evolution of these fluids assumes anisotropic character, so that not only the Hall current [1][2], but the anisotropic ion-slip current also must be taken into account.

In this paper it is considered an arbitrary region S, of boundary Σ, filled with an incompressible viscous and electrical conducting fluid, in isothermal inhomogeneous diffusion in S, in presence of an external uniform magnetic field. In the non relativistic case and under the usual hypotheses of the MFD [1][2], the basic equations of the anisotropic MHD governing the interactions between the fluid and the total magnetic field $\mathbf{H}$, are obtained by coupling the Navier-Stokes equations, including the Lorentz force, with the Maxwell's equations and a general constitutive equation (generalized Ohm's law) obtained by introducing the double tensor of the anisotropic electrical conductivity [1][2].

The mathematical model so obtained, in the isothermal case, is the following [1][2]:

$$\begin{cases} \dot{\mathbf{v}} = -\dfrac{1}{\rho_0}\nabla p^* + \nu\Delta\mathbf{v} + \dfrac{\mu_e}{\rho_0}\nabla\times\mathbf{H}\times\mathbf{H} \\ \dot{\mathbf{H}} = \mathbf{H}\cdot\nabla\mathbf{v} + \eta\Delta\mathbf{H} + \beta_1\nabla\times(\mathbf{H}\times\nabla\times\mathbf{H}) + \\ \qquad + \beta_2\nabla\times[\mathbf{H}\times(\mathbf{H}\times\nabla\times\mathbf{H})] \\ \nabla\cdot\mathbf{v} = 0 \\ \nabla\cdot\mathbf{H} = 0 \end{cases}$$

where the over dot means material time derivative, $(\mathbf{v}, \mathbf{H}, p^*)$ are velocity, magnetic, pressure fields, respectively. The constant positive transport parameters ν, μ_e, ρ_0, η are viscosity, magnetic permeability, density and magnetic viscosity; β_1 and β_2 are Hall and ion-slip coefficients which take into account of the anisotropy of the fluid. Moreover, to these equations, suitable initial and boundary conditions must be appended.

Methods

The nonlinear stability problem of a motion in S, in L^2-norm is investigated. By the direct Liapunov method and on introducing a suitable "energy partition functional", an asymptotic stability estimate for steady flows, useful for the application to the stability of anisotropic MHD planar Couette flows, with variable as well as constant total magnetic field is obtained.

The nonlinear stability of these flows, owing the high nonlinearities which are present in the problem, was studied only *with respect to 1-D laminar perturbations* in [12] and reported in [13] - chapter VIII, 8.3.

The stability problem to 3-D perturbations was open.

Therefore, recently, in the paper [14] the nonlinear stability to 3-D perturbations for these MHD Couette flows arising in a plane layer embedded in a coplanar magnetic field has been investigated. Here we show one of the results there obtained.

Results

By applying suitably the global stability estimate, it has been proved that for a class of Couette flows with suitable magnetic boundary conditions, asymptotic stability can be obtained also with strong values of the Hall and Ion-slip dimensionless parameters β_H and β_I, according with the conditions

$$\begin{cases} \dfrac{\beta_I}{\beta_H} > 1 \\ \beta_I > 1 \end{cases}$$

Therefore

"high levels of anisotropic currents can assure stability as well."

Conclusions

Using the direct Liapunov functional method we have investigated on the non linear stability of the hydromagnetic Couette-Poiseuille flows in L^2-norm, to 3-D perturbations, in presence of MHD anisotropic Hall and Ion-slip currents by the direct Liapunov method and we have proved that high levels of the anisotropic currents can assure asymptotic stability.

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