

MODULATED TAYLOR-COUETTE FLOW: ONSET OF SPIRAL MODES

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Abstract

The onset of spiral modes as a result of resonant dynamics in temporally modulated Taylor-Couette flow is explored via Floquet analysis.

Introduction

Modulated Taylor-Couette flow has been a paradigm problem for the parametric control of flow instability by temporal forcing. Both experiments and theoretical investigations (Donnelly *et al.*, 1962; Donnelly, 1964; Carmi & Tustaniwskyj, 1981; Walsh *et al.*, 1987; Walsh & Donnelly, 1988; Barengi & Jones, 1989; Murray *et al.*, 1990; Barengi, 1991; Ganske *et al.*, 1994) have shown that under differing combinations of mean state and temporal forcing, the onset of instability may be enhanced or delayed compared to the unforced system. Both may be desirable in different application settings. Regardless of the particulars of the temporally forced Taylor-Couette problem, there has been an overwhelming discussion in the literature concerning discrepancies between experiments and stability analysis at the lower end of the range of forcing frequencies considered. A number of possible explanations have been put forward, e.g. nonlinear transitions due to low frequency forcing allowing eigenmodes to grow beyond linear limit, and imperfections (Barengi & Jones, 1989; Barengi, 1991), but none of these have been systematically investigated and shown to be able to account for the discrepancies. On the theoretical side, it is apparent that for most studies only axisymmetric disturbances have been considered, and yet a number of experiments (Tennakoon *et al.*, 1997) have observed the onset of instability to non-axisymmetric modes in some parameter regimes.

Of all the variants, in this paper we consider the linear stability of the flow between concentric cylinder with the inner cylinder rotating at a constant angular velocity and the outer cylinder with angular momentum which is sinusoidal about a zero mean. For this configuration, Donnelly's group (Walsh & Donnelly, 1988) found that the modulation of the outer cylinder

delayed the onset of instability to higher rotation rates of the inner cylinder as compared to the zero-modulation case. Theoretical treatments of this flow (Carmi & Tustaniwskyj, 1981; Barengi & Jones, 1989; Murray *et al.*, 1990) have been unable to obtain reasonable agreement with the experiments in the low frequency large amplitude modulation regime. Here, we present a Floquet analysis of the time-periodic basic state that considers axisymmetric and nonaxisymmetric perturbation modes, and allows for variable axial wavenumbers; these two feature have not been considered in all previous studies. The non-axisymmetric perturbations lead to Neimark-Sacker bifurcations, Hopf bifurcations from limit cycles, and then the time integration for the monodromy matrix needs to allow for quasiperiodic bifurcated states.

Problem description

We consider the flow between two concentric cylinders, of infinite length and inner and outer radii r_i and r_o . The inner cylinder is rotating constantly, at Ω_i rad/s, while the outer cylinder oscillates about a zero mean at $\Omega_o = \epsilon\Omega_i \cos\omega t$ rad/s. The governing parameters are the inner Reynolds number, $Re_i = \Omega_i r_i (r_o - r_i) / \nu$, the radius ratio $\eta = r_i / r_o$, the forcing amplitude, ϵ , and the forcing frequency, ω , where ν is the kinematic viscosity.

The basic state, which can be found in terms of Kelvin functions (e.g., see Murray *et al.*, 1990), is time period. Its stability is determined by Floquet analysis, where the Navier-Stokes equations are linearized about the basic state, the linear system with a complete basis for general perturbations are evolved for a forcing period, and the eigenvalues of the resulting monodromy matrix are examined. Change of stability occurs when one or more eigenvalue has unit modulus; this can occur in three generic ways. The critical eigenvalue μ crosses the

unit circle through $+1$, a synchronous bifurcation; μ crosses through -1 , a subharmonic bifurcation. The third way is for a pair of complex conjugate eigenvalues to cross, $\mu_{1,2} = e^{\pm i\theta_0}$, a Neimark-Sacker bifurcation. The numerical Floquet analysis used here follows that of Marques & Lopez (1997).

Results

Here we present results for three gap ratios, $\eta = 0.88$ and 0.719 which have been considered previously by others, and 0.5 . Some salient features that will be discussed include: (i) at forcing frequencies near the natural frequency of the $m = 1$ spiral mode in the $\epsilon = 0$ unmodulated case leads to the change from synchronous to subharmonic bifurcation. This is a well-known result (although, that it occurs when there is a resonance with the unmodulated spiral modes has not been previously noted). For the axisymmetric perturbations, this codimension-2 point, at which the switch occurs, occurs at a much higher Re_i than for ω away from this resonance; this leads to a wedge-like region of stabilization (to axisymmetric perturbations), also previously discussed in the literature. (ii) In this wedge region, even though the $m = 0$ mode is stabilized, the non-axisymmetric spiral modes are destabilized. This is clearly seen in the critical Re_i versus ω figures. With small gap ($\eta = 0.88$ and 0.719), these Neimark-Sacker curves have many strong resonance points, from which complex dynamics can be expected in the nonlinear regime. (iii) There are several codimension-2 points in these resonance regions. In generic two-parameter (e.g. Re_i and ω) discrete-time systems (e.g. the Poincaré map corresponding to strobing at the forcing period), there are eleven types of codimension-2 points that can occur, four of these have yet to be treated theoretically, and are present in our resonance regions (Kuznetsov, 1998). These are

1. $\mu_1 = +1, \mu_2 = -1$;
2. $\mu_1 = +1, \mu_{2,3} = e^{\pm i\theta_0}$;
3. $\mu_1 = -1, \mu_{2,3} = e^{\pm i\theta_0}$;
4. $\mu_{1,2} = e^{\pm i\theta_0}, \mu_{3,4} = e^{\pm i\theta_0}$.

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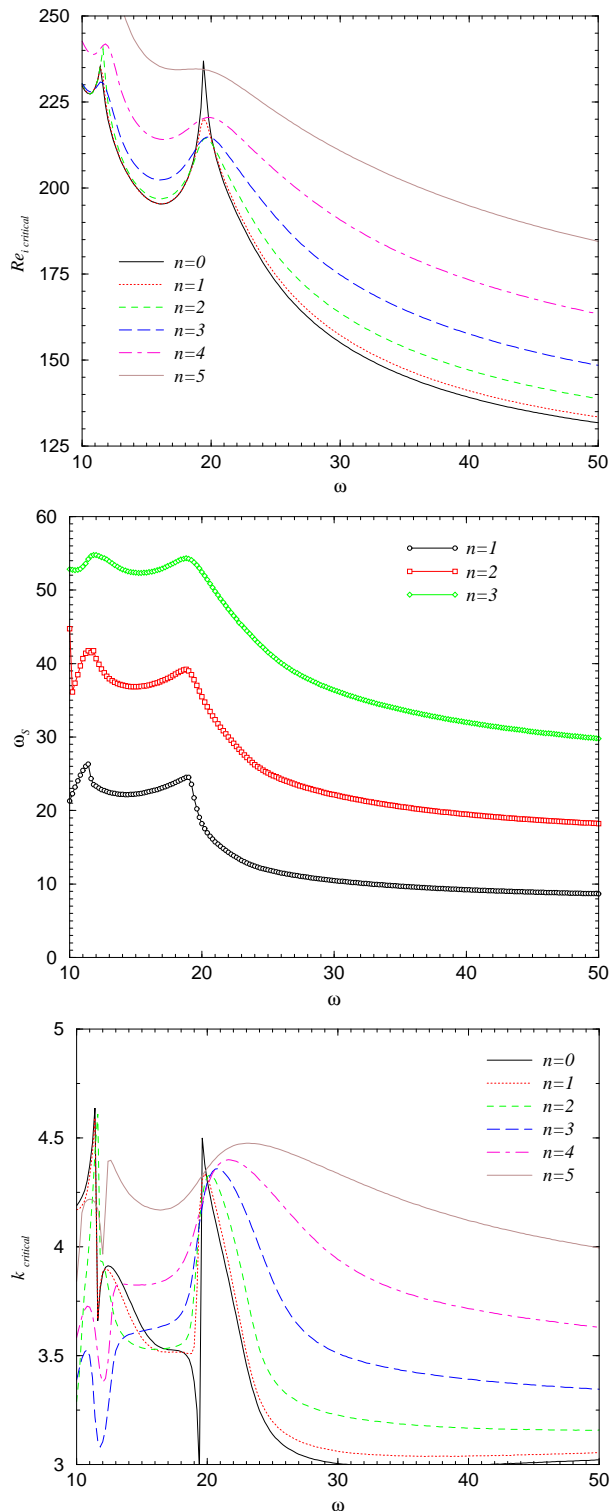


Figure 1: Variation of critical Re_i , ω_s , and k with ω for $\eta = 0.88$

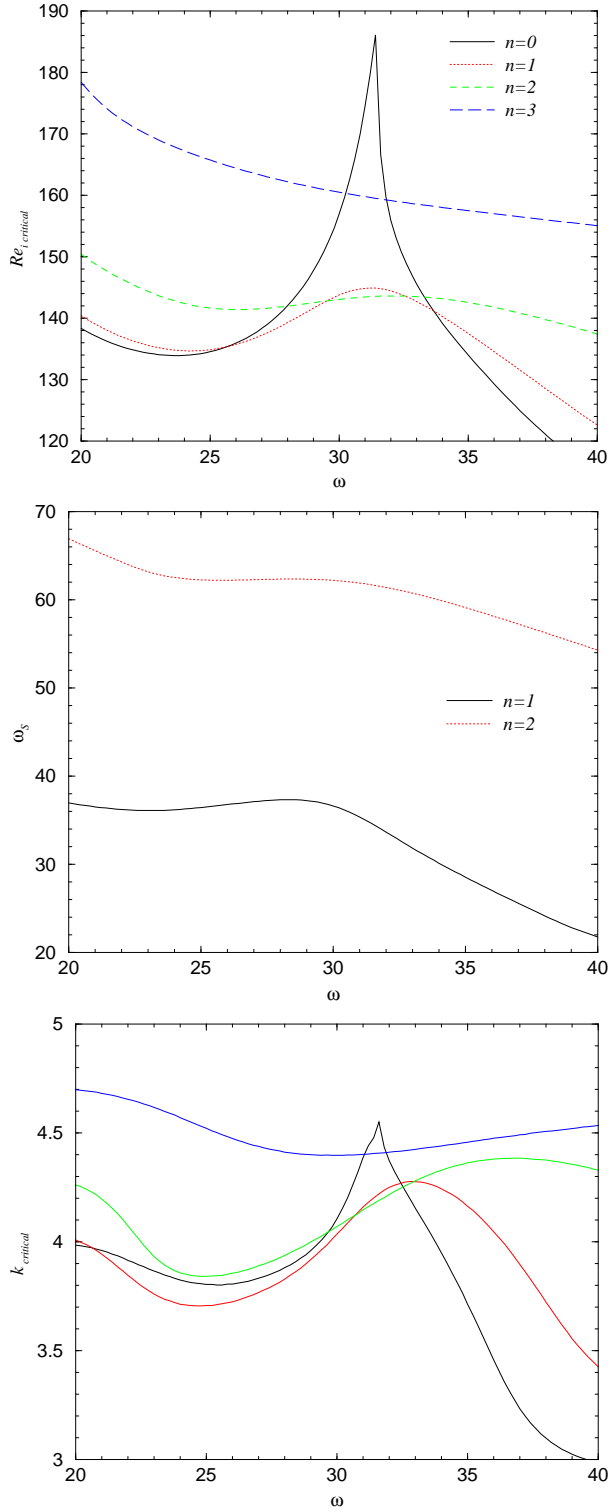


Figure 2: Variation of critical Re_i , ω_s , and k with ω for $\eta = 0.719$

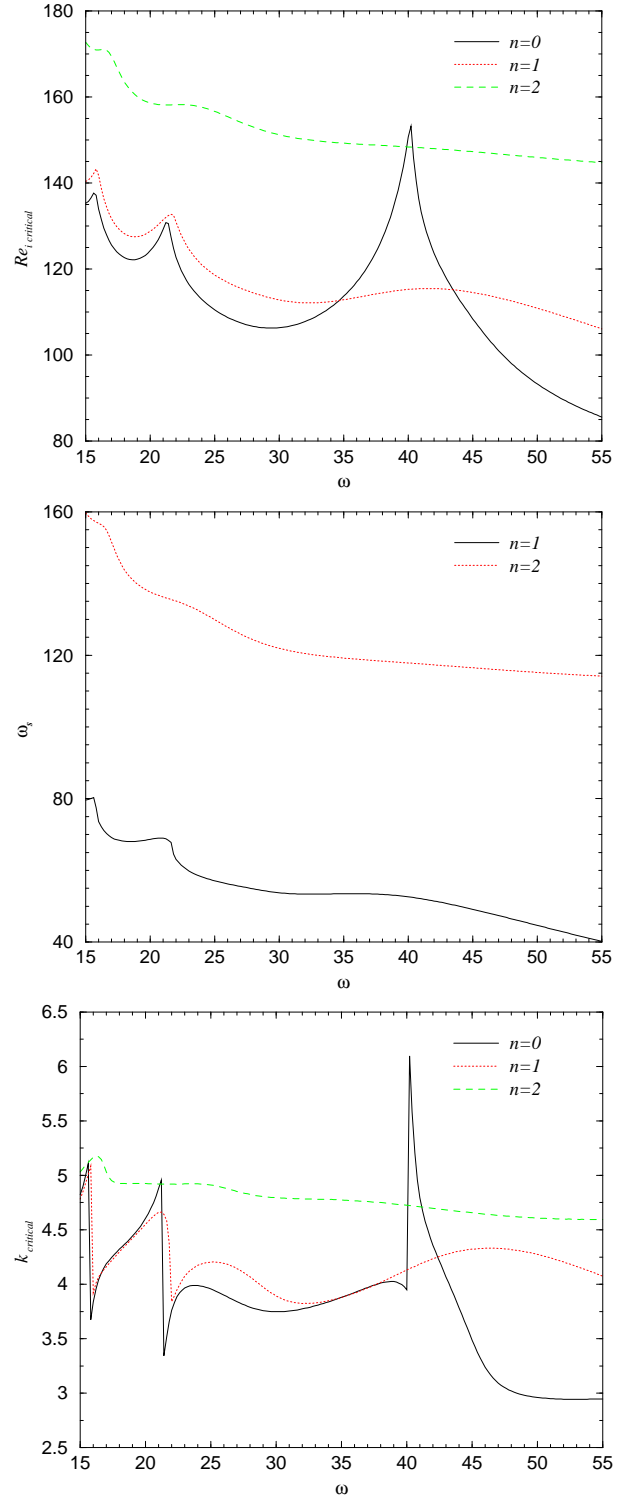


Figure 3: Variation of critical Re_i , ω_s , and k with ω for $\eta = 0.5$

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