12th International Couette-Taylor Workshop, September 6-8, 2001, Evanston, IL USA

STABILITY OF TAYLOR-DEAN FLOW OF BINGHAM FLUID

Blaise NSOM* and Hervé MANGEL

Université de Bretagne Occidentale. IUT de Brest. LUSIG/IUT. BP 45. Route de Kergoat. 29200 BREST (France)
*Corresponding author: B. Nsom, blaise.nsom@univ-brest.fr

Abstract

The flow of a Bingham fluid is considered between two rotating cylinders with a pressure gradient in the tangential direction. In order to complete the marginal stability study of the Couette-Taylor flow of that complex fluid, existing in the literature [1], the effect of the yield stress on the velocity field is presented, as well as the non-linear stability analysis. Then, the theoretical study of the Taylor-Dean configuration is performed. Introducing the rheological behavior law of the fluid in the conservation equations, the system governing the linear stability of the flow is produced with the small-gap approximation. Using the non slip cndition, an exact solution of that system is built in its general form. The Couette-Taylor flow of a Bingham fluid as well as the Taylor-Dean flow of a Newtonian fluid can be recovered with appropriate values of the parameters. The marginal stability surfaces of the Taylor-Dean flow are shown. This study was performed for environmental engineering skills, where the physical flowing material is the mud [2].

Steady circular flow

The rheological behavior of the Bingham model is governed by the following equation of state

$$s_{ij} = -p d_{ij} + g \frac{D_{ij}}{D'^{1/2}_{II}} + 2 m D_{ij}$$

where the g and m are constants, p the pressure and

$$D'^{1/2}_{II} = \frac{1}{2} D_{ij} D_{ij}$$

Introducing this law in the equations of conservation of the mass and the momentum, say

$$div(\vec{v}) = 0 \quad \text{and} \quad di\vec{v}(s) = r\vec{g}$$

r denoting the density of the mud, itself assumed incompressible, produces the following steady tangential flow for the Taylor-Dean configuration at low Reynolds number, with the following hydrodynamic field:

$$V = Ar + \frac{B}{r} + Kr.\ln(r) \quad ; \quad \frac{dp}{dr} = r\,\frac{V^2}{r}$$

where r is the radial coordinate in the gap; A and B, integration constants depending on the cylinders radii and tangential speeds, and on the two parameters K and N, characterizing the pressure gradient and the visco-plasticity of the mud. They are respectively defined as:

$$K = \frac{1}{2m}\left(\frac{\partial p}{\partial q}\right)_0 - \frac{g}{m} \quad ; \quad N = \frac{g}{m\Omega_1}$$

N is called the Bingham number characterizing the visco-plastic effects.

Equation for the marginal stability and limiting cases

If a stationnary perturbation with the classical axisymmetry and axial periodicity features is superimposed to this basic flow, the following equation of motion can be stated for the disturbance, after linearization:

$$[D^6 - A_4 D^4 + A_2 D^2 - A_0 - Bx]v = 0$$

where the A_i's denote polynomials of (6-i)th order for the axial wave number a; the coefficient B is proportional to the product Ta^2, T being the Taylor number, while x and v are the non dimensional radial coordinate and tangential velocity.

An exact solution of this differential equation was found using an integral representation, without assuming trial functions. The solution has the form of six infinite integrals which rapidly converge. Applying the non slip condition for the vortices on the cylinders walls leads to an homogeneous algebraic system, using

$$v(x) = D^2 v(x) = D[D^2 - a^2]v(x) = 0 \quad \text{for} \quad x = 0 \quad \text{and} \quad x = 1$$

The axial component of the velocity field is derived from this solution, using the continuity equation and the radial one, using

$$-Ta^2 u = [D^2 - a^2(1 + P - Q)]v$$

where P is the modified Bingham number (proportional to N) and Q is proportional to the tangential pressure gradient.
If Q is assumed zero in this system, the Couette-Taylor flow of a Bingham fluid is recovered. The results obtained here are in good concordance with those of Graebel [2].
If P is assumed zero, the Taylor-Dean flow of a Newtonian fluid is recovered and the well known results concerning this configuration are obtained with an excellent accuracy [3], [4].

Results

The non-Newtonian effects on the velocity field at the occurrence of the instability in the Couette-Taylor configuration were considered first. It was found that, increasing P moves the location of the maximum values of the velocity components closer to the internal cylinder

Table 1: Location of the maximum values of the velocity components vs. the modified Bingham number

	$x(u_{\max})$	$x(v_{\max})$	$x(w_{\max})$
P=0.00	0.47	0.49	0.19
0.10	0.47	0.48	0.19
0.25	0.46	0.48	0.18
0.50	0.46	0.47	0.18
1.00	0.45	0.46	0.17
1.25	0.44	0.46	0.17
1.50	0.44	0.45	0.17

Then, a non linear stability analysis was performed. Using Stuart's method, an energy equation was built. With an appropriate form for the velocity field, the following Landau equation for the amplitude A of the Taylor vortices in a Bingham fluid to order s was stated:

$$\frac{dA^2}{dt} = sA^2 + 2A^4[-\frac{1}{r}(m + \frac{t_0}{I})][J]$$

In this equation, t_0 denotes the yield stress, I the shear rate and J an integral expression for the radial coordinate. This amplitude equation clearly points out the effect of the visco-plasticity on the development of the instability, as the yield stress appears in addition to the mud viscosity.
Relaxing now the tangential pressure gradient, the critical parameters characterizing the occurrence of the instability in the Taylor-Dean configuration for a Bingham fluid were computed.
Typical marginal stability surfaces were obtained. They provide the variation of the critical wave number and Taylor number with respect to the parametrers P and Q. Projecting them in the P=*const* or Q=*const* spaces quantifies the effective role of each of these parameters on the characteristics of the Taylor-Dean instability for a Bingham fluid.
Furthermore, a fruitful discussion can be performed on the superposition of diverse stabilizing effects in Couette-Taylor flow. In this aim, a single pertinent parameter R was shown to govern the occurrence of the Taylor-Dean instability in a Bingham fluid. Table 2 gives the variation of the critical wave number a' and critical Taylor number T' with respect to R.

Table 2: Variation of the critical wave-number a' and critical Taylor number T' vs. parameter R

R	a'	T'
0	3.12	3389
0.10	3.06	3911
0.25	3.00	4750
0.50	2.91	6293
1.00	2.75	9914
1.25	2.71	11984
1.50	2.65	14223

References

[1] Graebel, P.G., 1962, "The hydrodynamic stability of a Bingham fluid in Couette flow"International Symposium on 2nd order effects in elasticity, plasticity and fluid dynamics, Haifa, Pergamon (1964), pages 636-649.
[2] Johnson, A.M., 1970, "Physical processes in Geology", Eds. Freeman Cooper and Co., San Francisco (California)
[3] Nsom, B., 1996, "Stability of fiber suspension flow in curved channel", J. Phys. II France, 6:1483-1492.
[4] Nsom, B., 1998, "Stability of Taylor-Dean flow of fiber suspensions", Appl. Mech. Eng., 3(2):185-204.