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TAYLOR VORTICES IN WIDE SPHERICAL SHELLS

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Abstract

We consider spherical Couette flow for aspect ratios in the range 0.15<Beta<0.45, and numerically obtain the axisymmetric solutions. We then map out the bifurcation diagram of all the steady solutions, stable and unstable, as they depend on both the Reynolds number and the aspect ratio. Finally, we compute the time−dependent solutions as well, and explore the transitions between different solutions.

Introduction

A variant of the classic Couette−Taylor problem is *spherical Couette flow*, the flow between concentric differentially rotating spheres. This simple configuration yields an enormous variety of flow patterns and transitions, even when consideration is restricted to axisymmetric flows. In this work we will explore some of these flows numerically, with the goal of mapping out the bifurcation diagram as it depends on the Reynolds number and aspect ratio.

We will consider only the simplest possible SCF scenario, namely where the outer sphere is stationary and only the inner one rotates. The flow is then determined by two parameters, the aspect ratio

$$\beta=(r_o-r_i)/r_i$$

describing the geometry, and the Reynolds number

$$Re=\Omega r_i^2/\nu$$

measuring the rotation rate. The usual procedure, both experimental and numerical, is then to fix the aspect ratio and scan through some range of Reynolds numbers to see what flow patterns one obtains.

However, the bifurcation diagrams one obtains at different aspect ratios are often quite different. For example, Fig. 1 shows the results obtained by Mamun and Tuckerman [1] at Beta=0.154, and by Hollerbach [2] at Beta=0.336. Certain features, such as the turning point marking the lower limit of existence of the one−vortex state, are clearly present for both aspect ratios. (The nomenclature one−state, two−state, etc. for SCF refers to the number of vortices in a single hemisphere.) However, for other features, such as the two Hopf bifurcations, the connection is not so obvious.

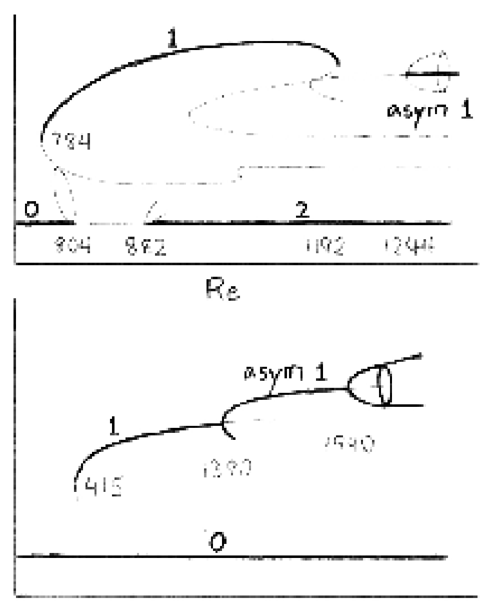

Figure 1: Bifurcation diagrams as functions of Re for Beta=0.154 (top) and Beta=0.336 (bottom).

In order to fully appreciate how the different solutions are related, one should therefore map out the complete two−parameter bifurcation diagram, i.e., to determine how the various states shown in Fig. 2 vary with both Reynolds number and aspect ratio. The numerical code used to accomplish this is the time−stepping code of Hollerbach [3], modified to do Newton−Raphson iteration as in Mamun and Tuckerman [1]. We began by mapping out all the steady solutions, unstable as well as stable. Then, having identified the locations of the various Hopf bifurcations, we are in a position to look for time−dependent solutions as well.

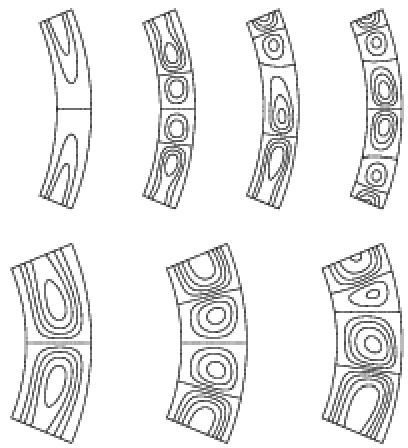

Figure 2: Examples of the four possible states at Beta=0.154, and the three possible states at Beta=0.336.

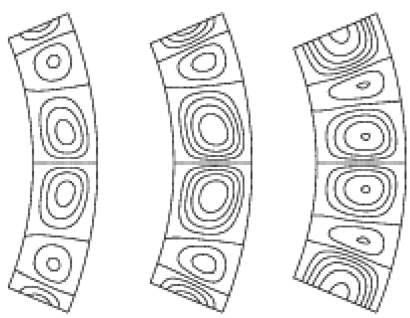

Figure 3: Examples of two−states at Beta=0.2, 0.25, 0.3.

Results

Here we will focus attention on one particular solution, namely the two−vortex state. As indicated in Figs. 1 and 2, this state exists at Beta=0.154, but not at 0.336. One interesting question might therefore be, how large* can one make the aspect ratio and still obtain this state? Fig. 3 shows two−states at Beta=0.2 to 0.3, which is near the upper limit. See also Fig. 7 of Junk and Egbers [4] for a picture of an experimentally obtained two−state at Beta=0.25.

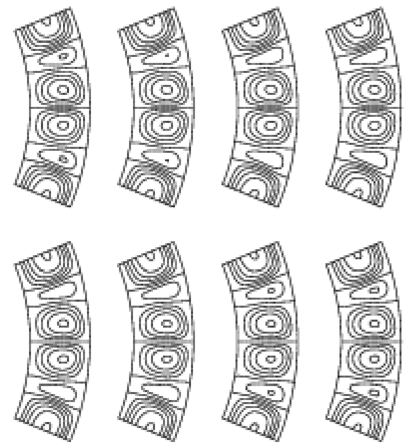

Figure 4: Eight snapshots in time of an equatorially symmetric state.

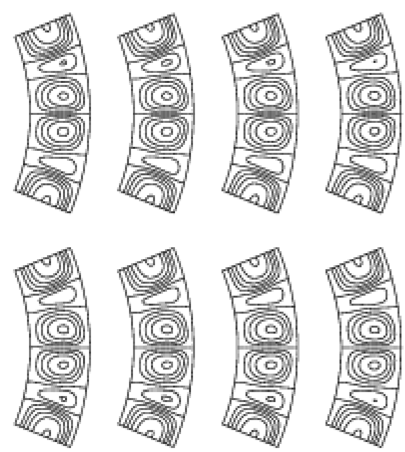

Figure 5: Eight snapshots in time of an equatorially asymmetric state.

Furthermore, if one now increases the Reynolds number, these solutions become time−dependent, exhibiting a very rich variety of behaviors. The basic time−dependence always consists of a fluctuation in the strength of the small vortices, much the same as the time−dependence of the asymmetric one−state discovered by Hollerbach [2]. However, because there are now two such vortices rather than just one, they could conceivably fluctuate either in phase − that is, the solution could continue to be equatorially symmetric, as in Fig. 3 − or out of phase − equatorially asymmetric. As shown in Figs. 4 and 5, both of these possibilities are indeed realized. The bifurcations leading to these two states will then be discussed in more detail. Finally, increasing the Reynolds number further still, these fluctuating solutions eventually collapse back to either the zero−state or the asymmetric one−state, as shown in Figs. 6 and 7.

* The very small aspect ratio limit, where one can obtain not only 2−states, but 3−states, 4−states, and indeed any number of n−states, is equally interesting, but will not be considered here.

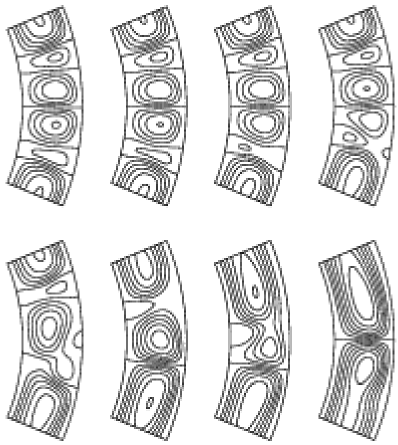

Figure 6: The collapse to the zero−state.

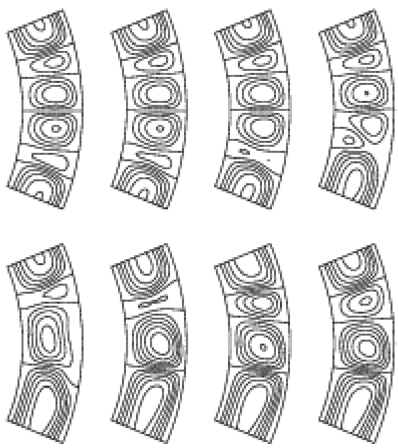

Figure 7: The collapse to the asymmetric one−state.

References

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