

LINEAR STABILITY OF RADIALLY HEATED TAYLOR COUETTE FLOW WITH RADIAL GRAVITY

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Abstract

In preparation for experiments that will simulate radial gravity using a ferrofluid in a magnetic field, the stability of radially heated circular flow has been computed for radial gravity of strength equal to axial gravity alone. Comparison is made to results with no radial gravity[1].

Introduction

The stability of circular Couette flow in the presence of a radial temperature gradient has been extended from the case of axial gravity [1]. to include an additional “radial gravity” term. That is, in addition to the buoyancy term $-\rho g \hat{z}$ we include a term $-\rho g_m f(r) \hat{r}$ where g_m is the effective gravitational acceleration in the radial direction. Physically this body force arises in experiments using ferrofluids in the annular gap whose inner cylinder surrounds a stack of equally spaced disk magnets [2]. The functional form $f(r)$ of the radial dependence of this force is found to be approximately

$$f(r) = \frac{K_1(\kappa r)}{K_1(\kappa r_1)},$$

where K_1 is the modified Bessel function of the first kind, κ is the axial wavenumber of the periodic variation of the magnetic field, and r_1 is the radius of the inner cylinder. Interestingly, even though the magnetic field B varies sinusoidally with axial position, the body force determined by the gradient of the magnitude of B is nearly independent of z for a well-designed laboratory experiment.

Methods

A standard linear stability approach using a code originally developed to explore isothermal counter-rotating circular Couette flow [3] was employed. An extension of this code to include radial buoyancy largely followed the radially heated Couette flow analysis of Ali & Weidman[1]. In addition to the Grashof number Gr , the Taylor number Ta , the Prandtl number

Pr , and the radius ratio η , the parameter space now includes the magnetic radial gravity $\gamma = g_m / g$. Disturbances of the form $F(r)e^{i(kz+m\theta-ot)}$ were explored.

Results

Preliminary neutral stability curves in $Ta - Gr$ space calculated for $Pr = 4.35$ at $\eta = 0.6$ show the effect of γ : the upper curve in Fig. 1 for $\gamma = 0$ matches the results of Ali & Weidman while the lower curve shows the effect of radial gravity for $\gamma = 1$.

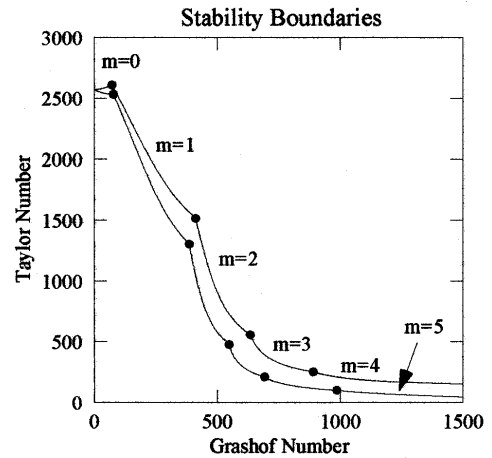


Figure 1: Stability boundaries for no radial gravity $\gamma = 0$ (upper curve) and for positive radial gravity $\gamma = 1$ (lower curve), with mode transitions (multicritical points) indicated by heavy dots. Positive values of γ and Gr indicate inwardly directed gravity and a higher

temperature on the inner cylinder than the outer cylinder, with a consequent outwardly directed buoyancy.

Conclusions

The tentative general conclusion is that radial buoyancy with $\gamma > 0$ destabilizes radially heated circular Couette flow. Also, except for the first mode transition from axisymmetric instability mode $m = 0$ to the first spiral mode $m = 1$, all mode transitions (indicated as heavy dots in Fig. 1) occur at lower Grashof numbers when $\gamma > 0$. Moreover, a transition to spiral mode $m = 5$ is observed at $Gr = 986$ for $\gamma = 1$ whilst the highest spiral mode for $\gamma = 0$ is for $m = 4$.

References

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- [3] Langford, W.f., Tagg, R., Kostelich, E. J., Swinney, H. L., and Golubitsky, M., (1988), "Primary instabilities in flow between counter-rotating cylinders," *Phys. Fluids*, 31:776-784.