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Symmetry breaking and restoring in perturbed plane Couette flow

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Abstract

Perturbed plane Couette flow containing a thin spanwise-oriented ribbon undergoes a subcritical bifurcation at Re = 230 to a steady 3D state containing streamwise vortices. This bifurcation is followed by several others giving rise to a fascinating series of stable and unstable steady states of different symmetries and wavelengths. First, the backwards-bifurcating branch reverses direction and becomes stable near Re = 200. Then, the spanwise reflection symmetry is broken, leading to two asymmetric branches which are destabilized near Re = 420. Time-evolution leads first to a metastable state whose spanwise wavelength is halved and then to complicated time-dependent behavior.

Introduction

Research on plane Couette flow has long been hampered by the absence of states intermediate in complexity between laminar plane Couette flow and time-dependent three-dimensional turbulence. Intermediate states can be created, however, if thin wire or ribbon oriented in the spanwise direction is inserted in an experimental setup [1,2]. No longer subject to Squire's theorem, this perturbed configuration undergoes a bifurcation to a three-dimensional steady or quasi-steady state. The 3D states contain vortices oriented in the streamwise direction and of finite streamwise extent, localized around the ribbon. As the wire or ribbon radius is reduced, the Reynolds number threshold for the bifurcation and the streamwise extent occupied by the vortices increase, while the range of Reynolds numbers over which the 3D steady states exist decreases. Experimental [1,2] and numerical [3] results are available for a particular ribbon height $\rho = 0.086$. Our calculation shows that this configuration has a rich bifurcation structure, and admits many types of solutions, stable and unstable, and of different symmetries.

Methods

Steady states were computed by integrating the time-dependent Navier-Stokes equations using the spectral element code `Prism` [4] written by Henderson. The domain is $[-32, 32] \times [-1, 1] \times [-\lambda_c/2, \lambda_c/2]$ where $\lambda_c = 4.8$ is the numerically determined critical wavenumber. Periodic boundary conditions were imposed at $x = \pm 32$ and at $z = \pm\lambda_c/2$, and no-slip conditions at the channel walls $y = \pm 1$ and at the ribbon $x = 0$, $|y| \leq 0.086$. In the (x, y) directions, we use 24×5 elements, each of which is covered by a grid of 7×7 collocation points or interpolating polynomials. Smaller elements are used near the ribbon. In the z direction, 32 Fourier modes or gridpoints are used, i.e. a total of 143840 gridpoints or basis functions per field.

Results

Figure 1 shows E_{3D}, the energy in the z-dependent modes for all the steady states we have calculated for $Re < 500$, and serves as a bifurcation diagram. Each branch is distinguished by its symmetry. The geometry and basic 2D flow have O(2) symmetry in the spanwise direction z, i.e. rotations $z \to z + z_0$ and reflections $z \to -z$. In the (x, y) plane, there is centrosymmetry $(x, y) \to (-x, -y)$.

The 2D branch loses stability via a circle pitchfork bifurcation at $Re_{\mathrm{CP}_1} = 230$ which breaks the rotation symmetry in z. The bifurcation is subcritical, and so the 3D states created branch leftwards and are unstable; we cannot calculate them with the methods used here. These states have reflection symmetry in z and centro-symmetry in (x, y); we call them 3D symmetric states. The centro-symmetry can be visualized as follows: At the ribbon location at $x = 0$, four small vortices are present. The upper two vortices persist for $x > 0$, while the lower two persist for $x < 0$; see figure 2.

The 3D branch changes direction and is stabilized by a saddle-node bifurcation at $Re_{\mathrm{SN}_1} = 197$. Its stability is short-lived, however, lasting only until a pitchfork bifurcation at $Re_{\mathrm{PF}_1} = 201$. The pitchfork bifurcation creates new branches with only the pointwise symmetry $(x, y, z) \to (-x, -y, -z)$; we call these 3D asymmetric states.

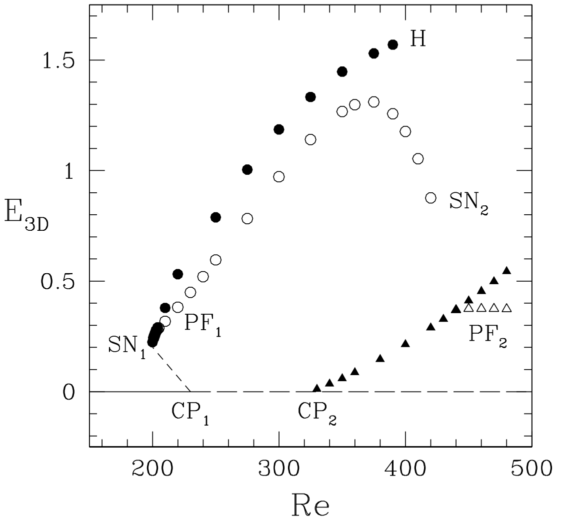

FIG. 1. Bifurcation diagram for perturbed plane Couette flow with ribbon height $\rho = 0.086$.
•Horizontal line: 2D states exist for all Re; they are stable for $Re < Re_{\mathrm{CP}_1} = 230$.
•Short-dashed line (schematic): unstable symmetric states with spanwise wavelength λ_c exist between $Re_{\mathrm{SN}_1} = 197$ and $Re_{\mathrm{CP}_1} = 230$.
•Filled circles: symmetric states with λ_c exist between $Re_{\mathrm{SN}_1} = 197$ and $Re_{\mathrm{H}} = 395$; they are stable between Re_{SN_1} and Re_{PF_1}.
•Hollow circles: asymmetric states with λ_c exist between $Re_{\mathrm{PF}_1} = 201$ and $Re_{\mathrm{SN}_2} = 420$.
•Filled triangles: symmetric states with $\lambda_c/2$ exist for $Re > Re_{\mathrm{CP}_2} = 330$.
•Hollow triangles: asymmetric states with $\lambda_c/2$ exist for $Re > Re_{\mathrm{PF}_2}$.

Figure 2 illustrates this symmetry breaking by showing two different velocity fields at $Re = 240$. The symmetric 3D field on the left has two different reflection symmetries, satisfying both $u(x, y, -z) = u(x, y, z)$ and $u(-x, -y, z) = -u(x, y, z)$. The asymmetric 3D field on the right satisfies only the single reflection symmetry $u(-x, -y, -z) = -u(x, y, z)$. Although the symmetric 3D branch is unstable, we can continue to calculate it by imposing reflction symmetry in z. It is further destabilized, however, by a Hopf bifurcation at $Re_{\mathrm{H}} \approx 395$, beyond which we have not followed it.

The asymmetric 3D branches change direction and are destabilized by a second saddle-node bifurcation at $Re_{\mathrm{SN}_2} \approx 420$. Surprisingly, time-dependent simulation at $Re = 450$ from an initial asymmetric state at $Re = 400$ leads to a metastable state with half the imposed wavelength of $\lambda_c = 4.8$, or equivalently, twice the critical wavenumber $\beta_c = 1.3$. The initial field at $Re = 400$ and the metastable state at $Re = 450$ are shown in figure 3. This transition is *symmetry-restoring* since the field is invariant under rotation in z by $\lambda_c/2$. Subsequent evolution from the metastable state is complicated and under investigation.

The metastable state is likely to be the $\lambda_c/2$ branch created from the 2D branch by a circle pitchfork bifurcation at $Re_{\mathrm{CP}_2} = 330$. Calculations show that it branches rightwards. Nevertheless, the $\lambda_c/2$ branch is necessarily unstable to wavelength doubling when it is created at Re_{CP_2} in a domain of size λ_c. (We calculate it in a domain of size $\lambda_c/2$.) This is because its parent 2D branch is already unstable to λ_c modes. Each of the two halves of the field is symmetric under reflection in z about its midplane $z = \lambda_c/4$ or $z = 3\lambda_c/4$. The $\lambda_c/2$ branch undergoes another pitchfork bifurcation at $Re_{\mathrm{PF}_2} = 440$, analogous to that undergone at Re_{PF_1}, creating branches which do not have this reflection symmetry. From figure 3, it can be seen that the vortices in the $\lambda_c/2$ field remain somewhat circular; their cross-channel height is reduced along with their spanwise extent. This could indicate that the streamwise velocity profile near the upper and lower walls is more stable than that near the center.

Acknowledgments

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References

[1] S. Bottin, O. Dauchot & F. Daviaud, 1997 “Intermittency in a locally forced plane Couette flow,” Phys. Rev. Lett., 79:4377–4380.
[2] S. Bottin, O. Dauchot, F. Daviaud & P. Manneville, 1998, “Experimental evidence of streamwise vortices as finite amplitude solutions in transitional plane Couette flow,” Phys. Fluids, 10:2597–2607.
[3] D. Barkley & L.S. Tuckerman, 1999, “Stability analysis of perturbed plane Couette flow,” Phys. Fluids, 11:1187–1195.
[4] R.D. Henderson & G.E. Karniadakis, 1995, “Unstructured spectral element methods for simulation of turbulent flows,” J. Comput. Phys., 122:191–217.

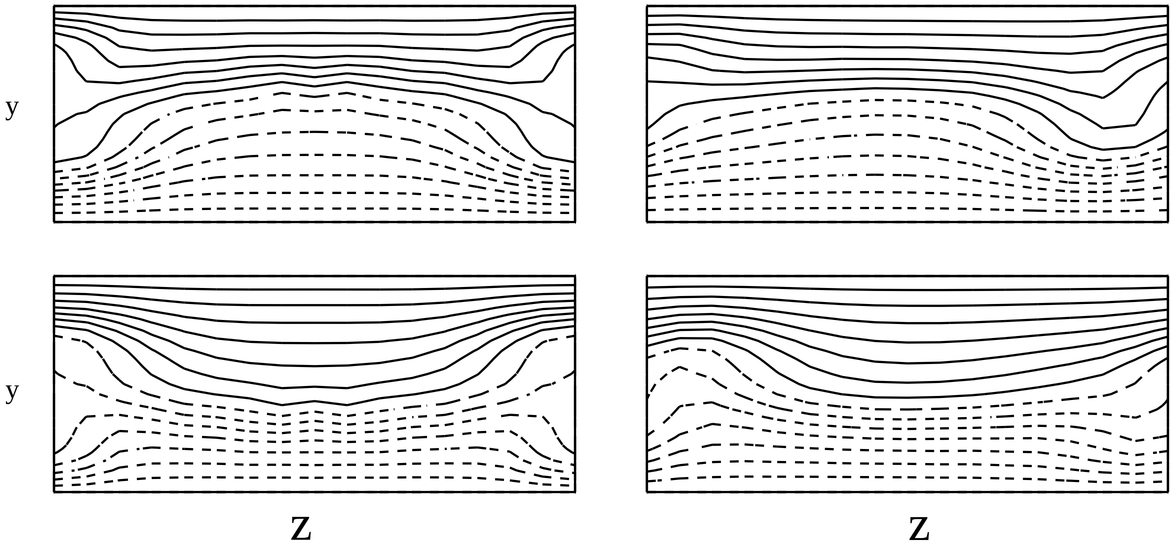

FIG. 2. Two velocity fields at $Re = 240$, illustrating breaking of reflection symmetry in z. Contours of streamwise velocity u are shown at $x = 2$ (above) and at $x = -2$ (below).
Left: State with reflection symmetry in z and centro-symmetry in (x, y). For $x = 2$, deformation of u contours shows that w velocity is upwards at mid-z. Thus vortex on left (right) is counter-clockwise (clockwise). For $x = -2$, the direction of w and vortex orientation are reversed.
Right: State with only the pointwise symmetry $(x, y, z) \rightarrow (-x, -y, -z)$.

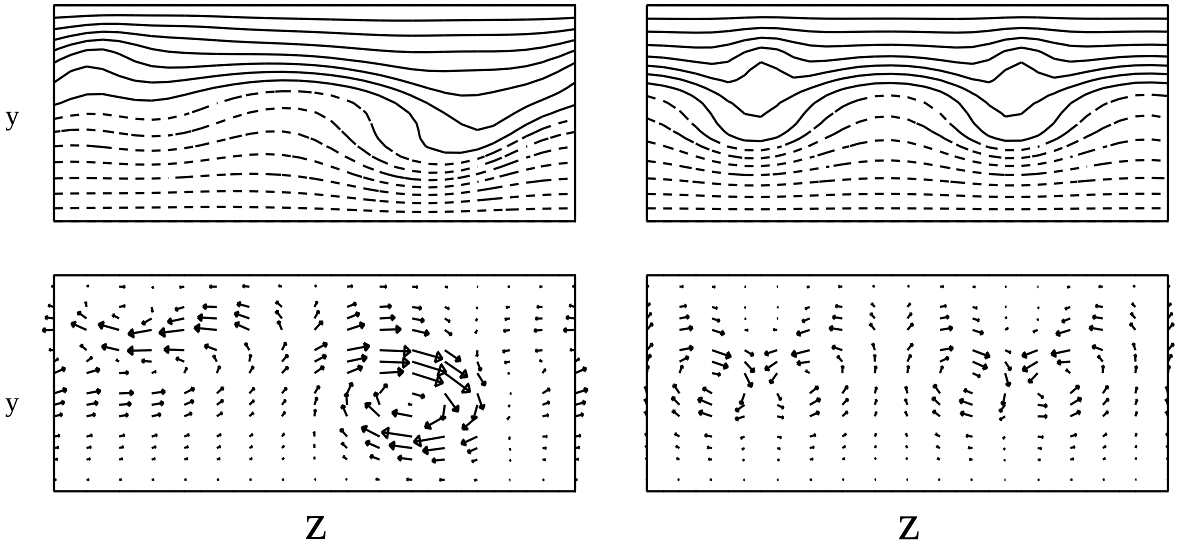

FIG. 3. Velocity fields at $Re = 400$ and $Re = 450$ illustrating symmetry-restoring transition. Contours of streamwise velocity u are shown above and (v, w) velocity field vectors are shown below, both at $x = 2$.
Left: Asymmetric state with spanwise wavelength $\lambda_c = 4.8$ at $Re = 400$. The asymmetry in z is very pronounced.
Right: Metastable state with spanwise wavelength $\lambda_c/2 = 2.4$ at $Re = 450$. The vortices occupy only the central portion in y.