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BANDED TAYLOR-COUETTE-POISEUILLE FLOW

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Abstract

It has previously been reported that several distinct flow patterns may result when two immiscible liquids undergo Taylor vortex flow in the annular gap between a fixed outer cylinder and a rotating concentric inner cylinder. One such flow pattern (banded flow) apparently arises when les dense disperse phase droplets migrate to Taylor vortex cores due to the centrifugal force associated with the rotation of the vortices. Using the hypothesis that centrifugally-driven droplet migration to vortex cores competes with turbulent diffusion, a simple criterion for predicting when banded flow patterns arise is developed and is shown to be consistent with numerical simulations and experiments.

Introduction

Several features of Taylor-Couette and Taylor-Couette-Poiseuille flow are attractive for chemical processing applications, although Couette devices are not yet widely deployed for these purposes. Some important examples of chemical processes that can be carried out to advantage in a Taylor vortex apparatus include emulsion polymerization [1], synthesis of silica particles [2], heterogeneous catalytic reactions [3,4], and liquid-liquid extraction [5,6,7]. More recently, several investigators have explored the utility of Taylor vortex devices as bioreactors and for filtration [8,9,10]. Most of these proposed or demonstrated uses for Taylor vortex flow involve multiphase and/or axial flow.

Much of what is known about emulsified liquid-liquid flows in devices with fixed outer cylinders has been reported within the past two decades. In particular, Joseph and co-workers reported a variety of flow patterns and instabilities including layered and banded Couette flows, rollers, coarse and foamy emulsions, and phase inversions [11,12]. More recently, Campero and Vigil surveyed hydrodynamic structures in liquid-liquid Taylor-Couette flow with a weak axial flow ($Re_z = 0.41 – 0.56$) [13,14]. At least three distinct structures were found including a translating banded pattern which appears to be caused by disperse phase droplet migration to vortex cores. In this report we provide evidence that this banded flow pattern is driven by centrifugal forces caused by the rotation of the vortices, and we develop a scaling relation that describes conditions leading to banded flows.

Mechanism of Banded Flow

We have carried out a series of experiments in a Taylor vortex reactor using water and kerosene as an immiscible fluid pair. By introducing catalytic amounts of surfactant, it is possible to vary the disperse phase (kerosene) droplet size while maintaining constant Taylor and axial Reynolds numbers. Photographs of these experiments clearly show that as the interfacial surface tension decreases (and consequently the disperse phase droplet size), the fluid undergoes a transition from banded flow to an apparently homogeneous emulsion. This change from banded flow at high surface tension to a spatially homogeneous emulsion at low surface tension suggests the following interpretation. When the surface tension is high (large droplets), the centrifugal forces arising from the rotation of the Taylor vortices (around an axis passing through their cores) cause the less dense disperse phase droplets to migrate to vortex cores. When the interfacial surface tension is very low, the droplets are sufficiently small so that they closely follow the movement of the surrounding fluid; the centrifugal force acting on the droplets is not strong enough to overcome turbulent fluctuations that disperse the droplets homogeneously in the gap.

If the above mechanism correctly accounts for the banded flow pattern, a criterion that estimates the transition between banded and homogeneous flow patterns can be obtained by comparing the characteristic time for a droplet released on the outer edge of a Taylor vortex to migrate to the core due to centrifugal forces, τ_c, with the characteristic time scale for a droplet to move from the vortex core to the vortex periphery due to turbulent diffusion, τ_t.

Using a steady-state radial force balance and assuming that the continuous phase fluid has no steady radial velocity component with respect to the rotational axis of the vortex, it can be shown that the ratio τ_c/τ_t is given by:

$$\frac{\tau_c}{\tau_t} = \frac{18\mu_c\mu_t}{\left(\rho_c - \rho_c\right)\rho_c d_p^2 u_\theta^2}, \qquad (1)$$

which should have a value of order unity near the transition between the banded and homogeneously dispersed states. Note that Eq. (1) depends upon both the physical dimensions of the Couette cell and the operating parameters indirectly through the disperse phase droplet diameter, d_p, the rotational velocity of the Taylor vortices (with respect to the votex core), u_θ, and the turbulent viscosity, μ_t. In the above expression μ_c represents the continuous phase viscosity and ρ_c and ρ_p represent the continuous and particulate phase densities, respectively.

Numerical Simulations and Experiments

In order to numerically explore the applicability of Eq. (1) for predicting the spatial distribution of a disperse phase with $\rho_p < \rho_c$ undergoing turbulent Taylor-Couette flow, axisymmetric simulations were performed using the computational fluid dynamics software FLUENT (version 5). The multiphase flow was modeled using FLUENT's implementation of the algebraic slip mixture (ASM) model, which can provide results comparable to a full two-phase model (particularly when the interaction of the phases is not well described), but with far less computational effort. The central feature of the ASM model is that rather than solving separate equations of continuity and momentum for each phase, a single momentum equation for the mixture is solved simultaneously with the equations of continuity for the mixture and for the disperse phase. In addition, the relative (slip) velocity between the disperse phase and the continuous phase is provided by an algebraic expression obtained from a force balance applied to a disperse phase particle. Turbulence is simulated by applying the Reynolds stress model to the mixture. A number of assumptions associated with the ASM model are employed including (a) particles are taken to be spherical, (b) the disperse phase particles have uniform and constant diameter, (c) no mass transfer is allowed between phases, (d) interfacial forces can be neglected, and (e) the individual phase pressures are taken to be equal.

In order to quantify the extent to which a flow is banded, we have calculated the ratio of the disperse phase volume fractions in the vortex core and in the reactor feed, α_c/α_f. By using various inner cylinder speeds, droplet sizes, fluid compositions, and axial flow rates, we determined the relationship between the vortex core volume fraction enhancement and the time scale ratio τ_c/τ. As was expected, the data lie on a universal curve such that α_c/α_f rapidly decreases to an asymptotic value of unity as the ratio τ_c/τ_t is increased from zero to one.

Photographs of the distribution of kerosene droplets in water in a series of experiments carried out using identical conditions (interfacial surface tension, fluid composition, axial Reynolds number, etc.) but with increasing inner cylinder rotation rates provide further, albeit qualitative, evidence for the validity of Eq. (1) as an index for predicting the "bandedness of the flow".

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