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Centrifugal Instabilities in lid-driven Cavity Flows

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Abstract

The flow of one-sided lid-driven cavities is a fundamental problem in fluid mechanics and has been treated numerically and experimentally by a large number of investigators. In the present paper we numerically calculate, for the first time, the linear stability boundaries of the two-dimensional cavity flow with respect to three-dimensional perturbations for several aspect ratios. Four critical instability branches arise, depending on the aspect ratio. By an energy analysis we claim that all instability branches are of centrifugal type.

Introduction

The first numerical investigation of the linear stability of the one-sided lid-driven cavity (fig. 1) is due to Ramanan and Homsy [6]. They predicted a critical mode with a wave number $k \approx 2$ for a cavity with unit aspect ratio. This result was questioned by Kuhlmann *et al.* [4] who provided evidence, using increasing grid sizes, that the neutral stability boundary of this particular mode does not converge to the result of [6]. The first reliable result for a neutral Reynolds number has been given by Ding and Kawahara [2, 3]. They found a a short wavelength mode of $k = 7.4$ and speculated this mode to be critical. More recently, Albensoeder *et al.* [1] also considered modes with higher wave numbers and found that the critical mode has a the wave number $k = 15.43$.

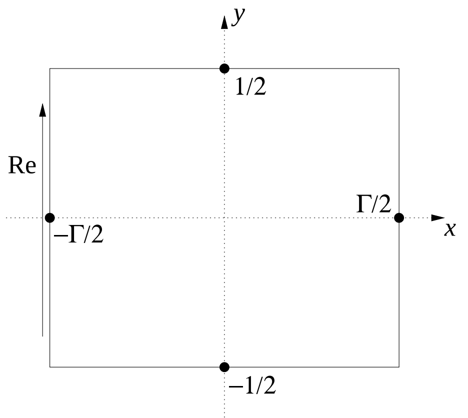

Figure 1: Geometry of the lid-driven cavity.

Methods of investigation

The basic two-dimensional flow $(\vec{u}_0, p_0)$ is calculated by solving the steady incompressible Navier–Stokes equations for a viscous fluid with kinematic viscosity ν. A finite-volume technique in primitive variables on a staggered grid with refinement near the boundaries is used. The resulting nonlinear equations are solved by Newton–Raphson iteration. To calculate the three-dimensional linear stability we consider normal modes $(\vec{u}, p) = (\vec{u}_0, p_0) + [\hat{\vec{u}}(x, y), \hat{p}(x, y)]e^{\sigma t + i(kz - \omega t)}$. The resulting generalized eigenvalue problem is solved by inverse iteration. While the typical grid resolution is 141×141, convergence test were made by comparison with 71×71 grid points. Lengths, velocities, pressure, and time are scaled by h, ν/h, $\rho\nu^2/h^2$, and h^2/ν, respectively, where h is the height of the square cavity. The non-dimensional parameters of the problem are the Reynolds number $\mathrm{Re} = Vh/\nu$ and the aspect ratio $\Gamma = d/h$ with V being the lid velocity and d the width of the cavity.

Results

Figure 2 shows the calculated neutral Reynolds numbers Re_n, wave numbers k, and frequencies ω as functions of the aspect ratio Γ. It turns out that four different modes may become critical, depending on the aspect ratio. Two modes in the intervals $\Gamma \in [0, 0.888]$ and $\Gamma \in [1.163, 1.207]$, respectively, are oscillatory, while the other neutral modes are steady. The critical mode for $\Gamma \to 0$ scales asymptotically like $\mathrm{Re}_c^* = \Gamma \mathrm{Re}_c(\Gamma \to 0) \approx 275$, $k_c^* = \Gamma k_c(\Gamma \to 0) \approx 5.1$ and $\omega_c^* = \Gamma^2 \omega_c(\Gamma \to 0) \approx 124$, where Re_c^*, k_c^*, and ω_c^* represent quantities scaled with the width d instead of the height h.

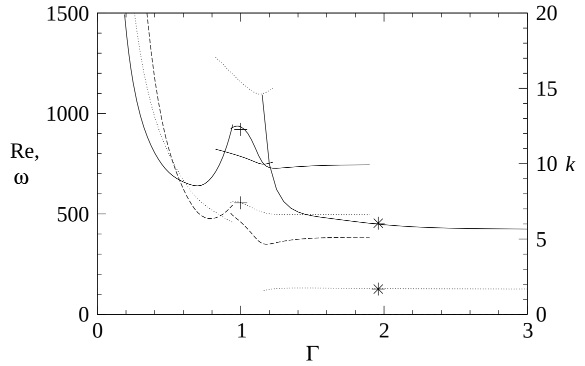

Figure 2: Neutral Reynolds numbers Re_n (solid lines) as functions of the aspect ratio Γ. The dotted and dashed lines denotes the wave number k and the oscillation frequency ω of the neutral mode, respectively. Previous results of [3] and [4, 5] are indicated by (+) and (∗), respectively.

For $\Gamma \in [0.888, 1.163]$ and $\Gamma \in [1.207, \infty]$ the critical modes are stationary. The asymptotic critical Reynolds number and wave number for infinitely wide cavities are $\mathrm{Re}_c(\Gamma \to \infty) \approx 420$ and $k_c(\Gamma \to \infty) \approx 1.685$.

For a detailed analysis of the instability process an *a posteriori* energy analysis [1] has been carried out. It turned out that all instabilities are caused by centrifugal effects. This conclusion is supported by application of the criterion of Sipp and Jacquin [7] for centrifugal instabilities.

Conclusion

The linear stability problem of the two-dimensional flow in one-sided lid-driven cavities has been solved for various aspect ratios. All critical modes are of centrifugal type. The asymptotic scaling for $\Gamma \to 0$ and $\Gamma \to \infty$ has been established..

Acknowledgement

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