

FLOW INSTABILITIES IN THE TWO-SIDED LID-DRIVEN CAVITY

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Abstract

The flow in rectangular cavities driven by two facing side walls is investigated numerically and experimentally for the case when the walls move steadily with equal speed in opposite directions. A linear stability analysis for the infinite system predicts different types of three-dimensional instabilities, depending on the aspect ratio and caused by either centrifugal or straining effects. The nonlinear behavior is explored by an experiment in which the separation of the moving walls is about twice the distance of the stationary walls. The flow field organizes into a row of robust rectangular steady cells which become time-dependent for increasing Reynolds numbers.

Introduction

The flow in the lid-driven cavity has been a popular benchmark test for Navier–Stokes codes since quite a time (see, e.g. [4]). Only recently, however, the linear stability of the single-lid-driven flow has been calculated numerically [2]. The extension of the classical lid-driven cavity problem to a system with two moving walls which face each other was originally motivated by the interest in flow topology [6] and mixing [8]. Kuhlmann et al. [7] were the first to investigate the nonlinear regime and found multiple two-dimensional states which were more systematically investigated by [1]. The former authors also reported the first observation of steady saturated flow states caused by the elliptic instability mechanism.

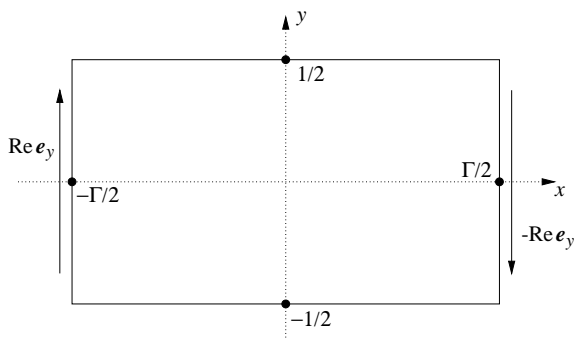


Figure 1: Cross-section of the cavity with coordinate system and boundary conditions (both walls move with equal speed).

Here we investigate the two-sided lid-driven

cavity problem (fig. 1) in which the two facing walls move in opposite directions and have the same velocity. Hence the Reynolds numbers based on the wall velocity and the length of the moving walls have the same magnitude ($Re_1 = Re_2$).

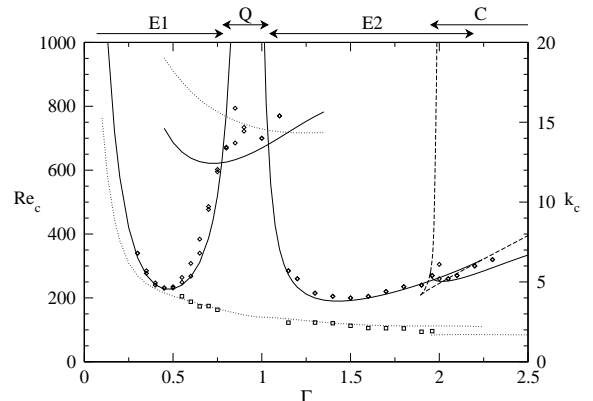


Figure 2: Neutral curves (full lines) and associated wave numbers (dotted lines) as function of the aspect ratio Γ . The diamonds and squares indicate experimental data. The labels on top denote the different sections of the critical curve.

Linear stability

The linear stability of the basic two-dimensional flow is calculated by a finite-volume method. Except for a small interval of the aspect ratios Γ , the basic two-dimensional flow is unique for the critical parameters. The critical Reynolds and wave numbers as a functions of the aspect ratio are shown in fig. 2. We find that all

instabilities are steady. A thorough discussion of the instability mechanisms will be presented elsewhere. According to our analysis the instability is of centrifugal type when the moving walls are far apart ($\Gamma \gtrsim 2$). It is of quadripolar-type [5] when $\Gamma \approx 1$, and of elliptic type, otherwise. The elliptic and quadripolar instabilities are caused by the strain field which is present in the flow when the basic vortex is deformed into elliptical and square shape, respectively.

Experiments

The apparatus for the experiments is a partially rebuilt version of the apparatus used by [7] with aspect ratio $\Gamma = 1.96$ and lengthwise aspect ratio $\Lambda = 6.55$. For this aspect ratio the instability is still due to the elliptic instability mechanism. The instability is supercritical and the flow pattern exhibits either four or five rectangular cells, each of which can arise in two variants which differ by a translation by half a wavelength. A typical five-cell flow is shown in fig. 3. Apart from end effects the steady flow exhibits a remarkable point symmetry with respect to the centers of each cell for the full range of Reynolds numbers for which it exists.

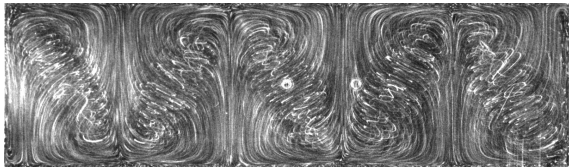


Figure 3: Streaklines of a fully developed steady five-cell flow for $Re = 700$ illuminated with a thick light sheet centered at $y = 0$.

Increasing Re beyond $Re = 875$ ($Re = 825$ for the four-cell flow) leads to flow oscillations via a supercritical instability. For oscillation amplitudes the cell structure of the flow remains intact. The visual observations and measurements with hot-film probes, flush-mounted to the bottom of the cavity at $x = 0$ and spanwise locations corresponding to the center of a cell and a boundary between adjacent cells, respectively, indicate that the point symmetry with respect to the steady state location of the cell centers is preserved at every instant of time. This symmetry is most likely responsible for the tricritical character of the secondary instability which is shown in fig. 4.

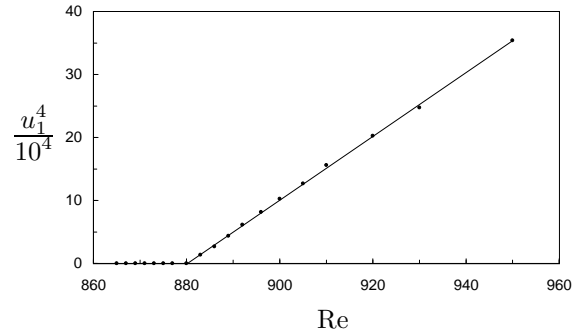


Figure 4: Fourth power of the basic harmonic (subscript 1) of the x component of the velocity field u (viscous scaling) measured by LDV at $(x, y, z) = (0, 1/4, 0)$ (above the center of the middle cell) in a five-cell mode as function of the Reynolds number Re .

Conclusions

The flow in a two-sided lid-driven cavity was investigated numerically and experimentally for anti-parallel wall motion. Different primary instabilities were found which have analogies in extended infinite systems as, e.g., the instability of homogeneous elliptic flow [9]. While the perturbations grow without bounds in infinite systems, the present cavity enables the observation of saturated nonlinear states. For $\Gamma = 1.96$ the saturated state consists of rectangular cells which may be called Λ -vortices, because they are created by the same (elliptic) mechanism and have the same structure as the well-known Λ -vortices which bifurcate from Tollmien–Schlichting waves [3]. The cellular Λ -vortices become unstable at higher Reynolds number to oscillations which bifurcate tricritically.

Acknowledgement

This work has been supported by DFG under grant numbers Ku896/5-2 and Ku896/8-1.

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