The circular Couette flow with a radial temperature gradient

Innocent Mutabazi*, Afshin Goharzadeh & Fabien Dumouchel

Laboratoire de Mécanique, Université du Havre
B.P. 540, 76058 Le Havre Cedex, France

*Corresponding author : mutabazi@univ-lehavre.fr

Abstract

We investigate experimentally spatio-temporal characteristics of patterns in the flow between two coaxial cylinders with an applied radial temperature gradient when the inner cylinder is rotating.

Introduction

The study of the influence of a radial temperature gradient on the stability of the circular Couette flow is important for different applications since ideal isothermal conditions are very difficult to achieve in practice. It is therefore important to know how a small temperature gradient applied on a flow can modify its stability. The circular Couette flow with a radial temperature gradient can modelize heat and mass transfer from rotating cylindrical bodies which are encountered in many applied systems such as cooling of rotating electrical motors or turbines, gas centrifuges, rotating heat exchangers,...[1]. While the stability of isothermal circular Couette flow has been subject of intense research [2,3], few work has been addressed on the stability of non-isothermal configuration, in particular little experimental work is available [1,4,5] and linear stability analysis studies have lead to contradictory results [6,7] because of a complexity related to the large number of control parameters which cannot be taken into account altogether.

Flow parameters

The inner cylinder has a radius a and rotates at the angular velocity $\Omega = 2\pi f_i$. The size of the annular gap occupied by the flow is d. The working fluid has the thermal expansion α, a kinematic viscosity ν and a thermal diffusivity κ. The rotation of the inner cylinder induces a circular Couette flow which is, according to ciruclar Rayleigh criterion, centrifugally unstable and gives rise to longitudinal steady vortices. The temperature gradient induces an axial flow, velocity profile of which possesses an inflexion point, then it may be unstable to transverse oscillatory perturbations according to Rayleigh-Fjörtfört criterion. The coupling between the two destabilizing mechanisms may induce new phenomena depending on different flow parameters. The control flow parameters are : the radius ratio $\eta = \dfrac{a}{a+d}$, the Prandtl number related to the fluid nature $Pr = \nu/\kappa$, the Taylor number related to the rotation and centrifugal force $Ta = \dfrac{\Omega a d}{\nu}\left(\dfrac{d}{a}\right)^{1/2}$, the Grashof number related to the temperature gradient $Gr = \dfrac{g\alpha \Delta T d^3}{\nu^2}$ where g is the gravity acceleration. For small temperature gradient, the base flow state has a two vorticity components : an axial one induced by cylinder rotation and an azimuthal component sometimes referred to as *baroclinic vorticity*, induced by the temperature gradient. For a given flow system (η and Pr fixed), the stability of the base flow is tested by varying the rotation frequency of the inner cylinder for a chosen value of the radial temperature gradient.

Experimental setup

The experimental setup consists of 3 coaxial cylindrical tubes: the inner tube is made of aluminium and has an external radius $a = 2$ *cm*, the intermediate and external tubes are made of glass and have inner radius $b = a + d = 2.5$ cm and $c = 5$ *cm*. In the inner tube and in the gap between the glass tubes, there are two water circulations from 2 cryo-thermostats with controlled temperature ($\pm 0.02°C$ from Bioblock Scientific). The inner tube is driven by a servomotor and rotates at the frequency f_i. We perform experiments for temperature differences in the range $\Delta T \in [-10,10]$°C. We add 1% Kalliroscope solution AQ-1000 to water in the annular gap for a better visualization with either a natural light or laser sheet along the axial direction. We have determined the critical frequency f_c at which the pattern occurs. Using a linear CCD with 2048 pixels, we have recorded at regular time interval, reflected off-pattern light intensity in order to obtain space-time diagrams from which we may deduce spatio-temporal properties of the pattern (frequency, wavenumber, defects,…).

Figure 1 : Photo of pattern spiral for
ΔT = 3°C near the onset.

Main results

1. The transition from laminar base flow occurs via a spiral pattern along the axial direction (Fig.1). The size of the pattern increases with the rotation velocity of the inner cylinder until the pattern fills the whole length of the flow at $f_i = f_c$. So the bifurcation occurs as a convective instability before an absolute instability sets in. For $\Delta T \geq 6°C$, the transition from base flow goes through an absolute regime instability. The stability curve $f_c(\Delta T)$ related to an absolute instability exhibits a small asymmetry between heating and cooling the inner cylinder (Fig.2).

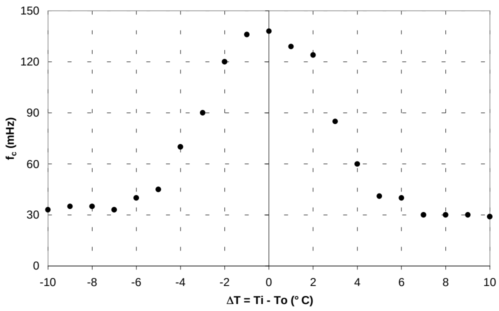

Figure 2 : Critical frequency f_c as function of the radial temperature gradient ΔT.

2. The stability curve (Fig.2) shows that small radial temperature (positive or negative) gradient destabilizes the flow.
3. The helicity of the spiral pattern depends on the rotation velocity vector and on the sign of the temperature gradient.
4. For $\Delta T = 3°C$, the space time diagrams of the spiral pattern, modulated spiral and modulated wavy pattern are given in Figure 3.a-c.
5. For fixed ΔT, the pattern wavenumber, the drift velocity depend on the rotation velocity while the inclination angle, depends only on the sense of rotation. In particular for $\Delta T = 3°C$ (Fig. 4-a,b), for $\varepsilon < 1$, the pattern is a spiral wavenumber and frequency of which increase with ε . For $\varepsilon = 1$, the spiral becomes modulated : its wavenumber pertains a sharp increase and a second low frequency occurs. For $1 < \varepsilon < 2.5$, the wavenumber of the modulated spiral weakly decreases, the two frequencies increase. For $\varepsilon = 2.5$, the spiral pattern becomes a modulated wavy pattern like that observed in isothermal Couette-Taylor flow. It has a constant wavenumber $q = 3.11$ and two frequencies in ratio around 3. The disappearance of inclination is related to the weakening of thermal effects compared to hydrodynamic effects.
6. For $\Omega = 0$, the instability occurs for $\Delta T = 15°C$ and gives rise to drifting pattern which becomes chaotic few percent above the onset.

Discussion

The coupling of rotation and radial temperature gradient induces a transition to oscillatory flow in form of spiral pattern since the axial velocity component has an inflexional point and is then unstable to transverse perturbations. Similar helicoidal vortex flow has been observed in the circular Couette flow with a small axial through-flow [9]. In the latter case, the instability may have two regimes : convective and absolute depending on the level of noise amplification induced by the through-flow. In our case, the axial flow has an ascending and descending parts which may amplify noise from top and bottom boundaries of the system.

The relative importance of buoyancy and rotation effects can be estimated using the Richardson number $Ri = \frac{Gr}{Re^2}$. For Ri close to unity, the two forces are of comparable magnitude, for isothermal Couette flow $Ri = 0$ and for natural convection flow $Ri = \infty$. The transition from modulated spiral to wavy modulated pattern occurs at $Ri = 0.03$ corresponding to weakening of thermal effects compared to rotation.

The traveling nature of the rolls and the asymmetry between negative and positive temperature gradient can be explained easily from a linear inviscid stability theory from which one deduce the frequency $f \propto Gr$ and a generalized Rayleigh discriminant given by

$$\Phi(r) = \Phi_0(r) + \Phi_{th}(r),$$

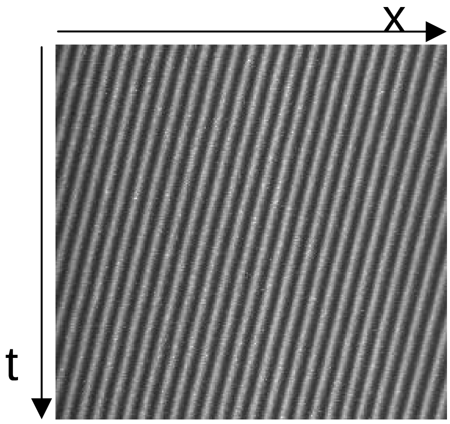

Figure 3-a : Space-time diagram of spiral pattern for f_{ic} = 0.085Hz (Ta_c = 29) and ΔT = 3°C (Gr = 1224).

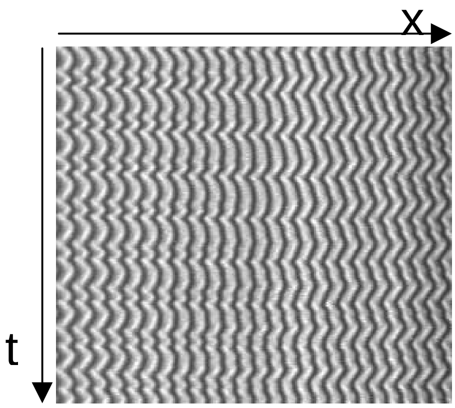

Figure 3-b : Space-time diagram of modulated spiral for f_i = 0.2 Hz (Ta = 68) and ΔT = 3°C.

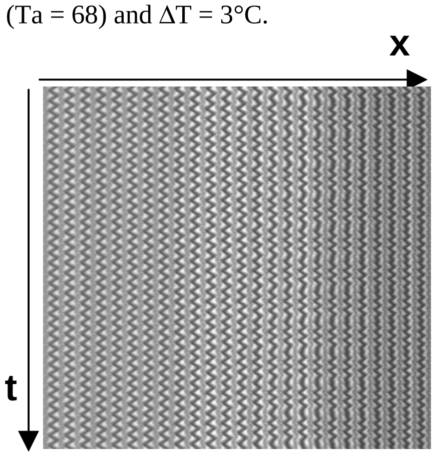

Figure 3-c : Space-time diagram of modulated wavy pattern for f_i = 0.4 Hz (Ta = 127), ΔT = 3°C.

where $\Phi_0(r)$ is the centrifugal Rayleigh discriminant and $\Phi_{th}(r)$ represents the thermal contribution given by

$$\Phi_{th}(r) = -\frac{AGr}{Re}(T - T_0)\Phi_0(r) - \frac{AGr}{Re}\frac{dT}{dr}\frac{V^2}{r}.$$

The parameter $A = \dfrac{\Omega a \nu}{g d^2}$ comes from the contribution of centrifugal acceleration in the buyoancy force, and it is responsible of the asymmetry between positive and negative gradient through the last term in $\Phi_{th}(r)$. This has been confirmed by results from linear stability analysis [7,10].

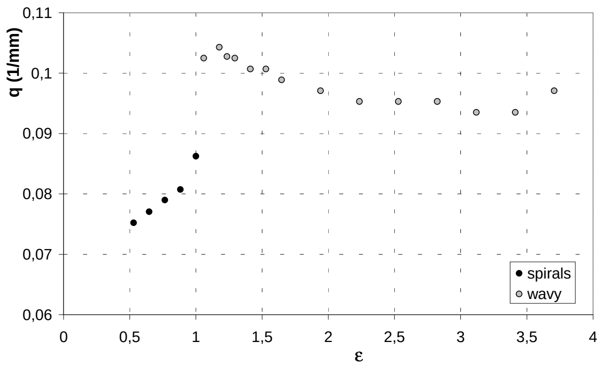

Figure 4-a : Variation of pattern wavenumber with the control parameter for ΔT = 3°C.

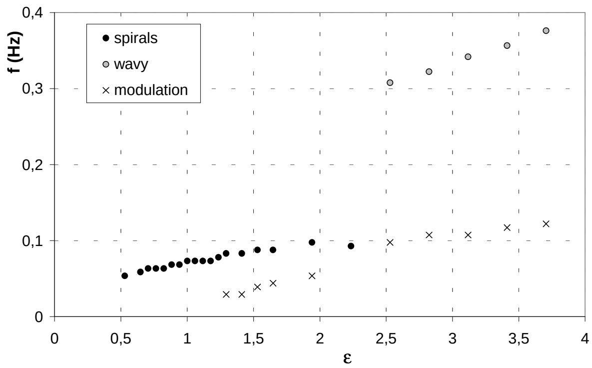

Figure 4-b : Variation of pattern frequency with the control parameter for ΔT = 3°C.

References

[1] P. Chossat & G. Iooss, *The Couette-Taylor problem*, Springer, New York (1995).
[2] C.D. Andereck, S.S. Liu & H.L. Swinney, "Flow regimes in a circular Couette system", *J.Fluid Mech*. **164**, 155-183 (1986).
[3] H.A. Snyder and S.K.F. Karlsson, "Experiments on the stability of Couete motion with a radial thermal gradient", *Phys. Fluids* **7**, 1696-1706 (1964).
[4] K.S. Ball & B. Farouk, "A flow visualization study of the effects of buoyancy on Taylor vortices", *Phys. Fluids* **A 1**, 1502-1507 (1989).
[5] K.S. Ball, B. Farouk & V.C. Dixit, "An experimental study of heat transfer in a vertical annulus with a rotating inner cylinder", *Int. J. Heat Mass Transfer* **32**, 1517-1527 (1989).
[6] M. Ali & P.D. Weidman, "On the stability of circular Couette flow with radial heating", *J. Fluid Mech*. 220, 53-84 (1990).
[7] J-C. Chen & J-Y. Kuo, "The linear stability of steady circular Couette flow with a small radial temperature", *Phys. Fluids* **A2**, 1585-1591(1990).
[8] R. Kedia, M.L. Hunt & T. Colonius, "Numerical simulations of heat transfer in Taylor-Couette flow", *J. Heat Transfer* **120**, 65-71 (1998).
[9] A. Tsameret & V. Steinberg, "Novel States in Couette-Taylor system with an axial flow", *Phys. Rev. E* 49, 1291 (1994).
[10] A. Bahloul, Etude numérique de la stabilité d'un écoulement de Poiseuille courbe avec un gradient radial de température, *Thèse de doctorat de l'Université du Havre*, 1997.

Acknoweledgements

This work was partly supported by a research grant from the Conseil Régional de Haute-Normandie. A.G. benefits from a scholarship of the French Research Ministry.